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QUANTITATIVE TECHNIQUES

By: Pritilata M.B.A.

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Sample Preview of the Solved Sample Question Papers

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QUESTION PAPER

(June – 2019)

(Solved)

QUANTITATIVE METHODS

Time: 3 Hours] *[Maximum Marks: 100*

Note: *Attempt questions from each section as per instructions given.*

SECTION - A

Answer the following questions.

Q. 1. A price discriminating monopolist operating in three market segments has demand function given by :

$P_1 = 63 - 4Q_1$

$P_2 = 105 - 5Q_2$

$P_3 = 75 - 6Q_3$

Where $Q_1 + Q_2 + Q_3 = Q$ (total output)

Its cost function is given by:

$C = 20 + 15Q$

Find the equilibrium quantities of Q_1, Q_2 and Q_3 and total profit and price charged in each market segment.

Ans. The monopolists revenues are:

$TR(Q_1) = 63Q_1 - 4Q_1^2$

$TR(Q_2) = 105\,Q_2 - 5Q_2^2$

$TR(Q_3) = 75Q_3 - 6Q_3^2$

The total costs are:

$TC = 20 + 15\,Q$

$= 20 + 15\,(Q_1 + Q_2 + Q_3)$

Implying that the marginal revenues

$MR(Q_1) = 63 - 8Q_1$

$MR(Q_2) = 105 - 10Q_2$

$MR(Q_3) = 75 - 12\,Q_3$

From the total cast,

$MC = 15.$

$MR = MC$

$MR(Q_1) \Rightarrow 63 - 8Q_1 = 15.$

$Q_1 = 6$

$MR(Q_2) \Rightarrow 105 - 8Q_2 = 15$

$Q_2 = 9$

$MR(Q_3) = MC = 75 - 12\,Q_3 = 15$

$\Rightarrow Q_3 = 5.$

So $P_1 = 63 - 24 = 39$

$P_2 = 105 - 45 = 60$

$P_3 = 75 - 30 = 45.$

So the monopolists profit at quantity is:

$= TR(Q_1 + Q_2 + Q_3) - TC.$

$= 999 - 320$

$= 679.$

Q. 2. *(a)* Write a linear first-order differential equation and work out its general solution.

Ans. Ref.: See Chapter-8, Page No. 44, 'Solving First Order Difference Equations'.

***(b)* Write the steps of solving the Harrod - Domar model of steady growth through differential equations.**

Ans. Ref.: See Chapter-7, Page No. 37, 'Harrod Domar Model'.

Q. 3. *(a)* If x is the sample mean, prove that expected value of $\bar{x}$, $E(\bar{x})$, equals the population mean μ.

Ans. Ref.: See Chapter-19, Page No. 147, 'Test Procedure Under Normality Assumption'.

***(b)* Describe the process of testing for a hypothesis as population proportion of a given attribute.**

Ans. Ref.: See Chapter-19, Page No. 146, 'Procedure of Testing Hypothesis'.

Q. 4. What is a Poisson Distribution? Bring out its important features. Give an example of a problem where you can use Poisson distribution.

Ans. The Poisson distribution is the discrete probability distribution of the number of events occurring in a given time period, given the average number of times the event occurs over that time period.

Conditions for Poisson Distribution:

- An event can occur any number of times during a time period.

Sample Preview of The Chapter

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QUANTITATIVE TECHNIQUES

INTRODUCTION TO DIFFERENTIAL CALCULUS

Functions, Limit and Continuity

1

INTRODUCTION

Mathematical Techniques are very useful in order to solve economic problems. Some concepts are related to function, limit, and continuity are expressed here. There are many ways to describe or represent functions: by a formula, by an algorithm that computes it, or by plotting graph. Making relationship between describing the problem and then interpreting it, is the main function of these mathematical expressions. In order to clear the concepts we will discuss some illustrations and also some practical exercises. The explanations given here will manage the understanding of different mathematical and statistical knowledge levels of students in each and every aspect.

CHAPTER AT A GLANCE

REVIEW OF THE BASIC CONCEPTS

Set

Sets are "collections". The objects "in" the collection are its members, e.g. we are all members of the set of all humans. There are sets of numbers, people, and other sets.

Example: $\{x : x \text{ is an even number}\}$

The set containing the even numbers (i.e. {0, 2, 4, ...})

FUNCTION

It is defined as the one quantity (the argument of the function, also known as the input) that completely determines another quantity (the value, or the output).

Domain: The set of all permitted inputs to a given function is called the domain of the function. Range: The set of all resulting outputs is called the image or range of the function.

Co-domain: The image is often a subset of some larger set, called the co-domain of a function. For example, the function $f(x) = x^2$ could take as its domain the set of all real numbers, as its image the set of all non-negative real numbers, and as its co-domain the set of all real numbers.

A function assigns exactly one value to each input of a specified type. The argument and the value may be real numbers, but they can also be elements from any given sets: the domain and the co-domain of the function.

Example of a function with the real numbers as both its domain and co-domain:

Function $f(x) = 2x$, which assigns to every real number the real number with twice its value.

In this case, it is written that $f(5) = 10$.

The notation $f : X \rightarrow Y$ indicates that f is a function with domain X and co-domain Y.

Variable: These are numerical values which changes with different mathematical operations. For example, *a, b, x, y*……

Continuous Variable: If y takes all prime numbers from a given number a to another given number b, then y is known as a continuous variable.

Interval: The value between two terminal values are known as interval. In the expression $(x + 3).(x + 5)$, the interval of x is –3 and –5.

Constant

The numerical value which never changes with change in mathematical operation. i.e. 5, 6, ½ π. . .

Absolute Value: The absolute value of a number is its distance from 0 on a number line.

For example, the number 9 is 9 units away from 0. Therefore its absolute value is 9.

In case of negative number, the absolute value is also positive. The number – 4 is still 4 units away from 0. The absolute value of – 4 is therefore positive 4.

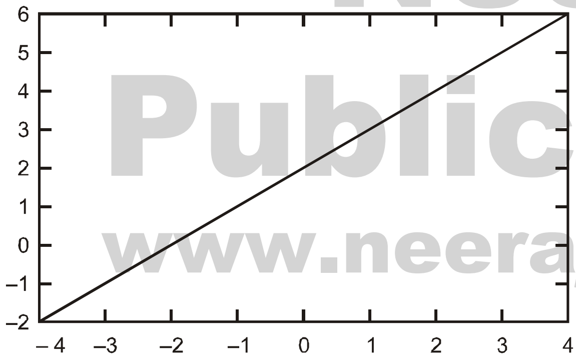
Examples:

$$|4| = 4$$
$$|-4| = 4$$
$$|4+3| = 7$$
$$|-4-3| = 7$$
$$|3-4| = 1$$
$$-|4| = -4$$
$$-|-4| = -4$$

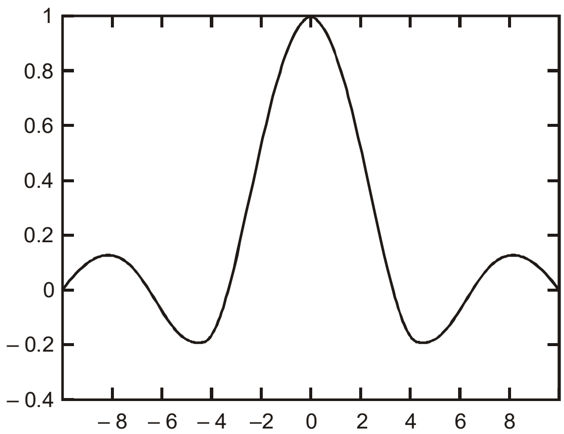
Graph of a Function

The symbol for the input to a function is often called the independent variable or argument and is often represented by the letter x or, if the input is a particular time, by the letter t. The symbol for the output is called the dependent variable or value and is often represented by the letter y. The function itself is most often called f, and thus the notation $y = f(x)$ indicates that a function named f has an input named x and an output named y.

Graph of the function $(x^2 - 4)/(x - 2)$



Graph of the function $\sin(x)/x$



Bounded Functions and their Bounds

A function f defined on some set X with real or complex values is called bounded, if the set of its values is bounded. In other words, there exists a real number $M < \infty$ such that

$|f(x)| \leq M$ for all x in X.

Sometimes, if $f(x) \leq A$ for all x in X, then the function is said to be bounded above by A. On the other hand, if $f(x) \geq$ for all x in X, then the function is said to be bounded below by B.

Monotone Function

A monotonic function (or monotone function) is a function that preserves the given order. A function f defined on a subset of the real numbers with real values is called monotonic (also monotonically increasing, increasing or non-decreasing), if for all x and y such that $x \leq y$ one has $f(x) \leq f(y)$, so f preserves the order. Likewise, a function is called monotonically decreasing (also decreasing or non-increasing) if, whenever $x \leq y$, then $f(x) \leq f(y)$, so it reverses the order.

If the order $\leq$ in the definition of monotonicity is replaced by the strict order $<$, then one obtains a stronger requirement. A function with this property is called strictly increasing. Again, by inverting the order symbol, one finds a corresponding concept called strictly decreasing. Functions that are strictly increasing or decreasing are one-to-one (because for x not equal to y, either $x < y$ or $x > y$ and so, by monotonicity, either $f(x) < f(y)$ or $f(x) > f(y)$, thus $f(x)$ is not equal to $f(y)$).

Inverse Function

f is a function from a set A to a set B, then an inverse function for f is a function from B to A, with the property that a round trip (a composition) from A to B to A (or from B to A to B) returns each element of the initial set to itself. Thus, if an input x into the function f produces an output y, then inputting y into the inverse function produces the output x, and vice versa.

Types of Function

1. **Algebraic Function:** An algebraic function is informally a function that satisfies a polynomial equation whose coefficients are themselves polynomials. For example, an algebraic function in one variable x is a solution y for an equation where the coefficients $a_i(x)$ are polynomial functions of x.

   A constant function is a function whose values do not vary and thus are constant. For example, if we have the function $f(x) = 4$, then f is constant since f maps any value to 4. More formally, a function $f : A \to B$ is a constant function if $f(x) = f(y)$ for all x and y in A.

2. **Non-Algebric Function:** A function which is not algebraic is called a transcendental function. i.e. $y = pqx$

CONCEPT OF LIMIT

Limit of a Function

The Limit of $f(x)$ as x approaches a is L:

$$\lim_{x \to a} f(x) = L$$

Example:

$$\lim_{x\to -2} f(x) \text{ where } f(x) = \begin{cases} -3x & \text{if } x \neq -2 \\ 1 & \text{if } x = -2 \end{cases}$$

$$\lim_{x\to -2} f(x) = \lim_{x\to -2} -3x$$

$$= -3 \lim_{x\to -2} x$$

$$= -3(-2) = 6$$

Right Hand Limit of a Function

The right-hand limit of $f(x)$, as x approaches a, equals L

$$\lim_{x \to a} f(x) = L$$

Left Hand Limit of a Function

The left-hand limit of $f(x)$, as x approaches a, equals M

$$\lim_{x \to a} f(x) = M$$

Examples of One-Sided Limit

$$\boxed{\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} 2x = 6}$$

$$\lim_{x\to 3^-} f(x)$$

$$\boxed{\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} x^2 = 9}$$

Functions Tending to Infinity:

If the extended real line R is considered, i.e. $R \cup \{-\infty, \infty\}$, then it is possible to define limits of a function at infinity.

If $f(x)$ is a real function, then the limit of f as x approaches infinity is L, denoted

$$\lim_{x \to \infty} f(x) = L,$$

if and only if for all $\varepsilon > 0$ there exists $S > 0$ such that $|f(x) - L| < \varepsilon$ whenever $x > S$.

Similarly, the limit of f as x approaches negative infinity is L, denoted

$$\lim_{x \to -\infty} f(x) = L,$$

if and only if for all $\varepsilon > 0$ there exists $S < 0$ such that $|f(x) - L| < \varepsilon$ whenever $x < S$.

For example

$$\lim_{x \to -\infty} e^x = 0$$

Fundamental Theorems on Limit

If c is any number, $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, then

(a) $\lim_{x\to a}(f(x) + g(x)) = L + M$

(b) $\lim_{x\to a}(f(x) - g(x)) = L - M$

(c) $\lim_{x\to a}(f(x) . g(x)) = L.M$

(d) $\lim_{x\to a}(f(x) / g(x)) = L/M, M \neq 0$

(e) $\lim_{x\to a}(c.f(x)) = c.L$

(f) $\lim_{x\to a} (f(x)n = Ln$

(g) $\lim_{x\to a} c = c$

(h) $\lim_{x\to a} x = a$

(i) $\lim_{x\to a} x^n = a^n$

(j) $\lim_{x\to a} \sqrt{f(x)} = \sqrt{L}, L > 0$

CONTINUITY

A function f is continuous at the point $x = a$ if the following are true:

(i) $f(a)$ is defined

(ii) $\lim_{x\to a} f(x)$ exists

If f and g are continuous at $x = a$, then

$f \pm g, fg$ and f/g $(g(a) \neq 0)$

are continuous at $x = a$

A polynomial function $y = P(x)$ is continuous at every point x.

A rational function $R(x) = P(x)/Q(x)$ is continuous at every point x in its domain.

Given $f(x) = 3x^2 - 2x - 5$,

Show that $f(x) = 0$ has a solution on $[1, 2]$.

$$f(1) = -4 < 0$$
$$f(1) = 3 > 0$$

$f(x)$ is continuous (polynomial) and since $f(1) < 0$ and $f(2) > 0$, by the Intermediate Value Theorem there exists a c on [1, 2] such that $f(c) = 0$.

Limits at Infinity

For all $n > 0$,

$$\lim_{x\to\infty} \frac{1}{x^n} = \lim_{x\to-\infty} \frac{1}{x^n} = 0$$

provided that $1/x$ n is defined.

CHECK YOUR PROGRESS

Q.1. Distinguish between variable and constants giving examples.

Ans. Variable changes with the change in numerical values whereas constant does not change with their numerical changes.

For Example: variables, *a, b, c, d, e…..*

Constant, π, 1/3, 4/9….

Q. 2. Point out the domain of definition of the following functions:

(i) **(cos *x* + sin *x*)/ (cos *x* – sin *x*) = *f*(*x*)**

Sol. Let (cos x + sin x) / (cos x – sin x) = 0

Domain = Any real value of x except $x = \dfrac{\pi}{4}$.

(ii) $\sqrt{x^2 - 5x + 6x} = f(x)$

Sol. Let $\sqrt{x^2 - 5x + 6x} = 0$

or, $x^2 - 3x - 2x + 6 = 0;$

or, $x(x-3) - 2(x-3) = 0;$

or, $(x-2)(x-3) = 0;$

or, $x = 2$ or, $x = 3$

Domain = All real value of x except $2 < x < 3$.

(iii) $f(x) = \sin^{-1} x$

Sol. Let $\sin^{-1}x = 0$,

Value of sin lies between –1 and 1

Domain = $-1 < x < 1$

(iv) **log (3*x* – 1) = *f*(*x*)**

Sol. Let log $(3x - 1) = 0$

or, $3x - 1 = 0$

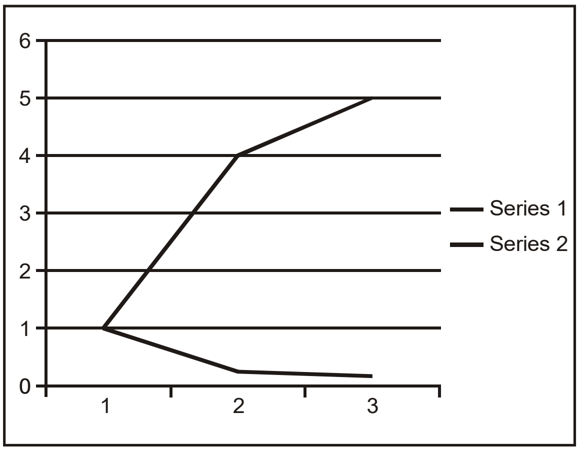
or, $x = 1/3$

Domain = value of $x > 1/3$.

Q. 3. $q = f(p) = 1/p$

Draw the graph of the above function. Show that this is a monotonically decreasing function. Also, show that the function has a lower bound (zero).

Ans.



Q. 4. $f(x) = x^3 + 2x$. **Find** $\dfrac{f(x+h) - f(x)}{h}$.

Sol. $\dfrac{f(x+h) - f(x)}{h} = \dfrac{f(x+h)}{h} - \dfrac{f(x)}{h}$

Putting the value of $(x + h)$ at place of x,

$$\frac{f(x+h)}{h} = \{(x+h)^3\}/h + \{2(x+h)\}/h$$
$$= (x^3 + h^3 + 3x^2 h + 3xh^2)/h + (2x + 2h)/h$$
$$= x^3/h + h^2 + 3x^2 + 3xh + 2x/h + 2$$
$$f(x+h) - f(x) = x^3/h + h^2 + 3x^2 + 3xh + 2x/h + 2 - x^3/h - 2x/h$$
$$= 3x^2 + 3xh + h^2 + 2.$$

Q. 5. Evaluate the following limits:

(i) $\lim_{x\to 0}\left(\sqrt{a+x} - \sqrt{a-x}\right)/x$

Sol. $\lim_{x\to 0}(\sqrt{a+x} - \sqrt{a-x})/x$

$$= \lim_{x\to 0}\left(\frac{\sqrt{a+x} - \sqrt{a-x}}{x} \cdot \frac{\sqrt{a+x} + \sqrt{a-x}}{\sqrt{a+x} - \sqrt{a-x}}\right)$$

$$= \lim_{x\to 0}\left(\frac{(\sqrt{a+x})^2 - (\sqrt{a-x})^2}{x} \cdot \frac{1}{\sqrt{a+x} - \sqrt{a-x}}\right)$$

$$= \lim_{x\to 0}\left(\frac{(\sqrt{a+x}) - (\sqrt{a-x})}{x} \cdot \frac{1}{\sqrt{a+x} - \sqrt{a-x}}\right)$$

$$= \lim_{x\to 0} \frac{2x}{x\sqrt{a+x} - \sqrt{a-x}}$$

$$= \lim_{x\to 0}\left(\frac{2}{\sqrt{a+x} - \sqrt{a-x}}\right)$$

- Events occur independently. In other words, if an event occurs, it does not affect the probability of another event occurring in the same time period.
- The rate of occurrence is constant; that is, the rate does not change based on time.
- The probability of an event occurring is proportional to the length of the time period. For example, it should be twice as likely for an event to occur in a 2 hour time period than it is for an event to occur in a 1 hour period.

The Poisson distribution is the discrete probability distribution of the number of events occurring in a given time period, given the average number of times the event occurs over that time period.

Features of a Poisson Distribution

- The experiment consists of counting the number of events that will occur during a specific interval of time or in a specific distance, area, or volume.
- The probability that an event occurs in a given time, distance, area, or volume is the same.
- Each event is independent of all other events. For example, the number of people who arrive in the first hour is independent of the number who arrive in any other hour.

Example: In the World Cup, an average of 2.5 goals are scored each game. Modeling this situation with a Poisson distribution, what is the probability that kk goals are scored in a game?

In this instance, λ = 2. 5. the above formula applies directly.

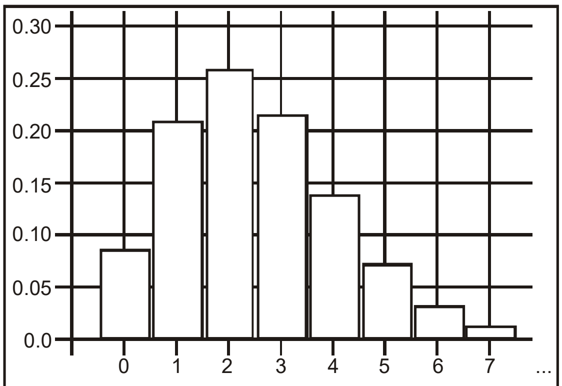
$$P(X = 0) = \frac{2.5^0 e^{-2.5}}{0!} \approx 0.082$$

$$P(X = 1) = \frac{2.5^1 e^{-2.5}}{1!} \approx 0.205$$

$$P(X = 2) = \frac{2.5^2 e^{-2.5}}{2!} \approx 0.257$$

$$P(X = 3) = \frac{2.5^3 e^{-2.5}}{3!} \approx 0.213$$

$$P(X = 4) = \frac{2.5^4 e^{-2.5}}{4!} \approx 0.133$$



The Poisson Distribution with λ = 2.5

There is no upper limit on the value of *k* for this formula, though the probability rapidly approaches 0 as *k* increases.

SECTION - B

Answer the following questions.

Q. 5. A linear programming problem is given as: max $z = 30x_1 + 50x_2$

subject to : $x_1 + x_2 \geq 9$

$x_i + 2x_2 \geq 12$

$x_1 \geq 0, x_2 \geq 0$

Find its optimal solution.

Ans. Max $z = 30x, + 50x_2$

Subject to $x_1 + x_2 \geq 9$

$x_1 + 2x_2 \geq 12$

$x_1, x_2 \geq 0$

Max $Z = 30x, + 50x_2 + OS_1 + OS_2 - MA_1 - MA_2$

$x_1 + x_2 - S_1 + A_1 = 9$

$x_1 + 2x_2 - S_2 + A_2 = 12$