I.G.N.O.U. *Based*

NEERAJ®

B.P.H.E.-101

ELEMENTARY MECHANICS

By: *Gajendra Nayal*

Question Bank** Cum **Chapterwise Reference Book
Including** **Many Solved Question Papers

NEERAJ PUBLICATIONS
(Publishers of Educational Books)
(An ISO 9001 : 2008 Certified Company)

Sales Office:
1507, 1st Floor, Nai Sarak, Delhi - 6
Ph.: 011-23260329, 45704411,
23244362, 23285501
E-mail: info@neerajbooks.com
Website: www.neerajbooks.com

MRP ₹ 300/-

Published by:

NEERAJ PUBLICATIONS

Sales Office : **1507, 1st Floor, Nai Sarak, Delhi-110 006**

E-mail: **info@neerajbooks.com**

Website: **www.neerajbooks.com**

Reprint Edition with Updation of Sample Question Paper Only	Typesetting by: Competent Computers	Printed at: Novelty Printer

CONTENTS

ELEMENTARY MECHANICS

Question Bank – (Previous Year Solved Question Papers)

S.No. **Chapterwise Reference Book** *Page*

REVISING PARTICLE KINEMATICS AND DYNAMICS

WORK, ENERGY AND ANGULAR MOTION

■ ■

Sample Preview of the Solved Sample Question Papers

Published by:

NEERAJ PUBLICATIONS
www.neerajbooks.com

QUESTION PAPER

(June – 2019)

(Solved)

ELEMENTARY MECHANICS

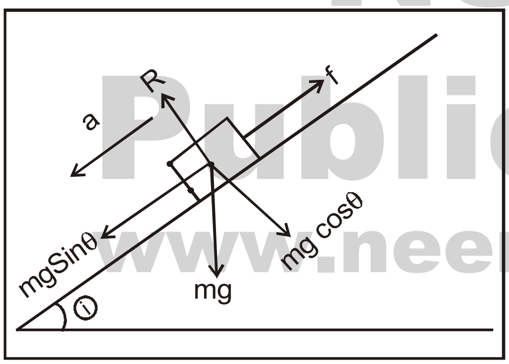
Time: $1^{1/2}$ hours] *[Maximum Marks : 25*

*Note: Attempt **all** questions. The marks for each questions are indicated against it. Symbols have their usual meanings. You may use log tables or calculators.*

Q. 1. Attempt the following parts:

***(a) (i)* A crate of mass 3.0 kg is moving down an inclined plane which makes an angle of 30° with the horizontal. If the coefficient of kinetic friction between the crate and the plane is 0.15, calculate the acceleration of the crate. Draw the freebody diagram. Take g = 10ms^{-2} .**

Ans. The free diagram is given below, Here, m = mass of crate, R = Normal reaction, F = Friction.



Given, m = 3kg, g = 10 m/s^2

Acceleration of crate, a = g ($\sin\theta$ – $\mu\cos\theta$)

$\Rightarrow$ a = 10 (Sin 30^o – 0.15 cos 30^o)

$$= 10\left(\frac{1}{2} - 0.15 \times \frac{\sqrt{3}}{2}\right) = 3.70 \text{ m/s}^2 .$$

***(ii)* A lift of mass 3000 kg moves 100m upwards at a constant speed in 25s. At what average rate does the force due to the cable do work on the work on lift?**

Ans. Let us assume that the lift is moving in the positive y-direction.

$$\vec{F} = [3000\text{ kg} \times 9.8\text{ ms–2}]\ \hat{j}$$

$$\vec{F} = (29400)\ \hat{j}$$

$$\vec{V} = \frac{100\text{m}}{25\text{ s}}\ \hat{j} = 4\text{ ms}^{-1}\ \hat{j}$$

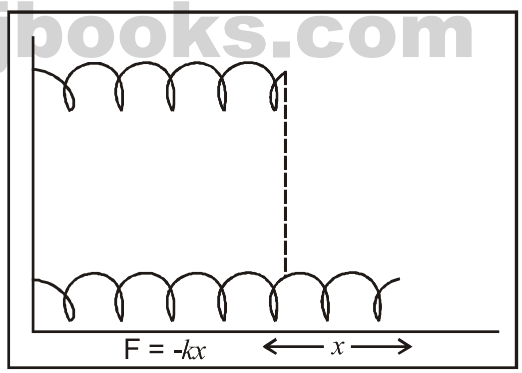
$$\therefore \quad P = \vec{F}.\vec{V}$$

$$= (29400)\ \hat{j} \times (4\text{ ms}^{-1})\ \hat{j}$$

$$= 11.76 \times 10^4\text{ W.}$$

***(b) (i)* One end of a spring is attached to a fixed wall and the other end to a block that is free to slide on a horizontal surface. Determine the work done by the spring force as the block is moved a distance d by stretching the spring. Is the spring force conservative?**

Ans.



By the above figure

$$F = -kx\ \hat{i}$$

and $$\overrightarrow{dr} = dx\ \hat{i}$$

$$\therefore \quad W = \int_0^d \vec{F}.\overrightarrow{dr}$$

$$= \int_0^d -kx\,dx = -\left[\frac{x^2}{2}\right]_0^d$$

$$= \frac{kd^2}{2}$$

Yes, the spring force is conservative.

***(ii)* A spaceship lifts off vertically from the Moon where the free-fall acceleration is 1.6 ms^{-2}. What is the weight of the astronaut on the Moon if her weight on the Earth is 600N ? Take g = 10ms^{-2} .**

Ans. Acceleration of spaceship on moon, $g^1 = 1.6$ ms^{-2}

Weight of the pilot on earth W = 735

Mass of pilot, $m = \frac{w}{g} = \frac{600}{10} = 60$ kg

Weight of the astronaut on moon,

$$W^1 = mg^1$$
$$= 60 \times 1.6$$
$$= 96\text{ N}.$$

***(c)* State the law of conservation of energy. An object of mass 50Kg is pushed from rest across a rough floor with a force of 200N. The coefficient of kinetic friction between the object and the floor is 0.20. Determine the speed of the object after it has travelled 40m. Take g = 10 ms^{-2}.**

Ans. According to the law of conservation of energy, energy can neither be created nor destroyed. It can only be transferred from one form to another.

Using Work – Energy Theorem:

$$W_{total} = \Delta K.E$$
$$W_1 = 200\text{ N} \times 40\text{ m} = 8000\text{ J}$$

The work done by force of friction:

$$W_2 = -(\mu k\text{ mg})d$$
$$= -(0.20 \times 50\text{ kg} \times 10\text{ ms}^{-2}) \times 40$$
$$= -4000\text{ J}$$
$$W_{Total} = W_1 + W_2 = 8000\text{ J} - 4000\text{ J} = 4000\text{ J}$$

Using work energy theorem we get

$$W = \frac{1}{2}\text{mVF}^2$$
$$= \frac{1}{2} \times 50 \times \text{VF}^2$$
$$\text{VF} = \sqrt{\frac{4000 \times 2}{50}} = 13.41\text{ ms}^{-1}$$

In the absence of friction, W = 8000 J, then

$$\text{VF} = \sqrt{\frac{8000 \times 2}{50}} = 17.88\text{ ms}^{-1}.$$

***(d)* A merry-go-round is accelerated uniformly from rest and attains an angular speed of 0.5 rad s^{-1} in the first 10s. If the net applied torque is 2000 Nm, calculate the moment of inertia of the merry-go round. What will the angular momentum of the merry-go-round be after 20s? Is the angular momentum conserved ? Explain.**

Ans. Given, angular speed, $w = 0.5$ rad s^{-1}

time, t = 10s

Torque, σ = 2000 Nm

We know that

$$T = I\alpha$$
$$\Rightarrow \quad T = \frac{I dw}{dt}$$
$$\Rightarrow \quad 2000 = I\,\frac{(0.5-0)}{10}$$
$$\Rightarrow \quad I = \frac{2000 \times 10}{0.5} = 40000\text{ kg m}^2$$

Angular speed after 205,

$$w = w_o + \alpha\,t = 0.5 + \frac{1 \times 20}{20} = 1.5\text{ rad } s^{-1}$$

Angular momentum

$$L = Iw$$
$$= 40000 \times 1.5$$
$$= 60000\text{ kg m}^2/\text{s}$$

Yes, angular momentum, remain conserved.

Q. 2. Attempt the following parts :

***(a) (i)* Derive the law of equal areas for a central force.**

Ans. Ref.: See Chapter-11, Page No.175, 'The Law of Equal Areas'.

***(ii)* A comet moves around the Sun in an elliptical orbit of eccentricity 0.9. If the length of the orbit's semi-major axis is 3.0×10^{12} m, calculate the distance of the comet from the Sun at perihelion and aphelion.**

Ans. From the formula for perihelion and aphelion

$r_p = a(1-e)$ and $r_a = a(1+e)$

Given, $r_a = 3 \times 10^{12}$ m and $e = 0.9$

So,

$$r_p = 3 \times 10^{12}(1-0.9) = 3 \times 10^{11}\text{ m}$$
$$r_a = 3 \times 10^{12}(1+0.9) = 5.7 \times 10^{12}\text{ m}.$$

***(b)* The position vectors of three particles of masses 1.0 kg, 2.0 kg and 3.0 kg are given by $(t+2t^2)\hat{i} + 3t\hat{j}$, $2\hat{i} + 6t^2\hat{j}$ and $(4t+1)\hat{i} + t^2\hat{j}$ respectively. Calculate the velocity and acceleration of the centre of mass of the system.**

Ans. The position vector of LM of three particles ($\vec{r}_{cm}$) will be given by :

Sample Preview
of
The Chapter

Published by:

NEERAJ
PUBLICATIONS
www.neerajbooks.com

ELEMENTARY MECHANICS

REVISING PARTICLE KINEMATICS AND DYNAMICS

1

Motion : An Introduction

INTRODUCTION

Motion is common to everything in the universe. We walk, run and ride a bicycle. Even when we are sleeping, air moves into and out of our lungs and blood flows in arteries and veins. Automobiles and planes carry people from one place to the other. The earth moves/rotates once every twenty four hours and revolves round the sun once in a year.

In physics, motion is a change in position of an object with respect to time and its reference point. Motion is typically described in terms of displacement. velocity, acceleration and time.

CHAPTER AT A GLANCE

WHAT IS MOTION?

In everyday life, we see some objects at rest and others in motion. Bird fly, fish swim, blood flows through veins and arteries and cars move. Atoms molecules, planets, stars and galaxies are all in motion. We often preceive an object to be in motion when its position changes with time. However, these are situations where the motion is inferred through indirect evidences. For example, we infere the motion of air by observing the movement of dust and the movement of leaves and branches of trees.

Most Motions are complex. Some objects may move in straight line others may take a circular path. Some may rotate and a few others may vibrate. There may be situations involving a combination of these. The motion of objects can be specified by words like position, distance, displacement, speed velocity and aceleration. Representation of motion is called kinematics.

MOTION IN A STRAIGHT LINE

In such a motion, the object moves along a straight line. For example, motion of a train along a straight railway track, a man walking on a level and narrow road, an object falling freely under gravity etc.

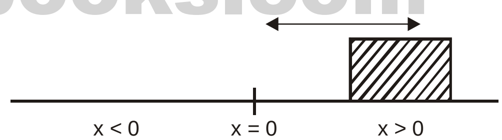
In kinematics we need to be able to have a way to describe the motion of the objects. We will be studying whether it's a car or an atom.

The most basic information you must have to describe the motion of an object is its position and the time it was in that position.

Position, Distance and Displacement

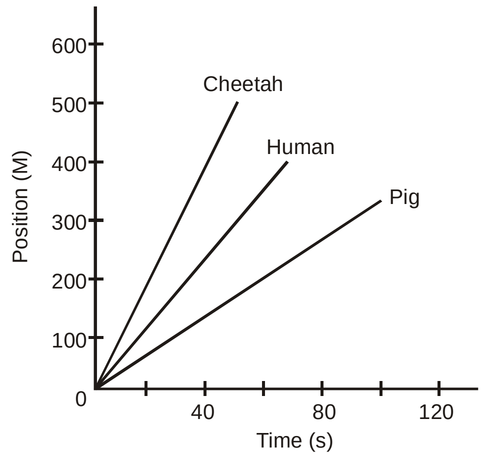
The position of an object is always taken from some reference point which is usually zero on the scale.

The position of an object along a straight line can be uniquely identified by its distance from a origin. The position is fully specified by 1 co-ordinate.



One Dimensional Position

The slope of the curve in the position versus time graph depends on the velocity of the object. See figures shown in below. After 10 seconds the cheetah has covered a distance of 310 m, the human 100 m and the pig 50 m. Obviously the cheetah has the highest velocity. A similar conclusion is obtained when we consider the time required to cover a fixed distance. The cheetah covers 300 m in 10 seconds the human in 30 seconds and the pig requires 60 seconds. It is clear that a steeper slope of the curve in the distance *(x)* vs time *(t)* graph corresponds to a higher velocity.



x vs t graphs for various creatures.

Although we use the words "distance" and "displacement" interchangeably in our routine language, they mean very different things in physics.

The distance between two objects is scalar, since it does not matter which direction you measure it from. For example: We are standing 2.3 m apart.

The displacement of an object is a vector, since you have to state the direction the object has travelled. For example: The car moved 2.56 km east.

The most simple formula for calculating the change in an object's position is

$$\Delta d = d_f - d_i$$

Where d_f represent final distance and d_i represent initial distance.

For example: A truck is passing a mark on the road that says 300 m, and then passes another one 10 second later that says 450 m. Determine the distance the truck moved.

Ans. $\Delta d = d_f - d_i$

$= 450 - 300$

$= 150\ m.$

CHANGES OF POSITION AND DISPLACEMENT

We are now to deal with displacement to change in position.

Position Vector

Specifying the position of an object is essential in describing motion. In one dimension some typical ways are.

x (t) is used to respresent position as a function of time. The position has only one component, so it effectively degenerates to a scalar co-ordinate it could be say a vector in the x-direction or the radial r-direction. Equivalent notation include:

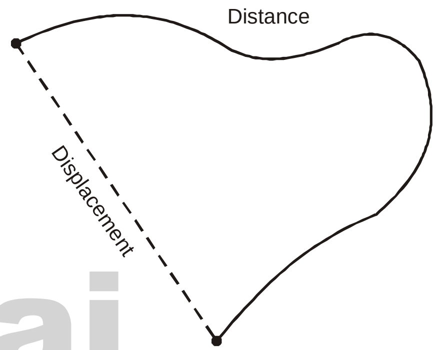
$x \equiv x \equiv x\ (t)$,

$r \equiv r\ (t)$,

$s \equiv s\ (t)$.

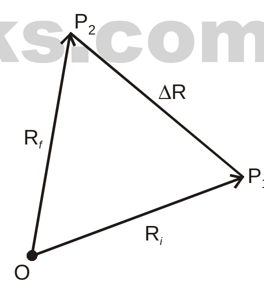
Displacement

A displacement is a vector that is the shortest distance from the initial to the final position of a point P. It quantifies both the distance and direction of an imaginary motion along a straight line from the initial position to the final position of the point.



Displacement versus distance travelled along a path. A displacement may be also described as a 'relative position': the final position of a point (R_f) relative to its initial position (R_i) and a displacement vector can be mathematically defined as the difference between the final and initial position vectors.

Displacement = $R_f - R_i = \Delta R$.



Displacement in terms of Co-ordinate and Unit Vector

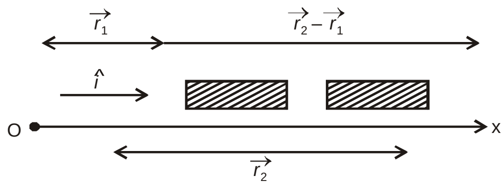
A position vector expresses the position of an object from the origin of a co-ordinate system. Now we define the positive x-axis with object moving direction in a straight line. Let us denote the unit vector along the positive x-axis by $\hat{i}$.

Let an object in motion be at origin O are x_1 and x_2 with respect at the instant t_1 and t_2. Let us denote the position vectors of the object with respect to O at these two instants by $\vec{r}_1$ and $\vec{r}_2$.

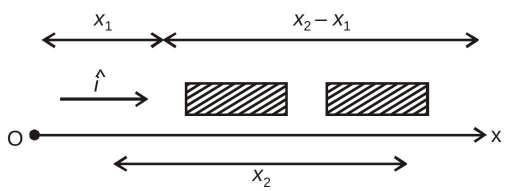
Thus, we define the position vectors $\vec{r}_1$ and $\vec{r}_2$ in terms of $\hat{i}$ as

$\vec{r}_1 = x_1\,\hat{i}$ and

$\vec{r}_2 = x_2\,\hat{i}$ ...(*i*)



Position vectors and displacement



Co-ordinate of objects in straight line motion. So, using equation (*i*), we denoted the displacement

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

As well as in terms of the co-ordinate and unit vector as

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$
$$= (x_2 - x_1)\,\hat{i}$$

So, we can express the positive *x*-axis to be along the magnitude of the displacement in a straight line given by:

$$\Delta x = (x_2 - x_1)$$

Their directions are along the *x*-axis.

SELF-ASSESSMENT QUESTIONS

Q. 1. In figure below *(a)*, *(b)* and *(c)*, which objects are at rest and which ones in motion, with respect to whom or what?

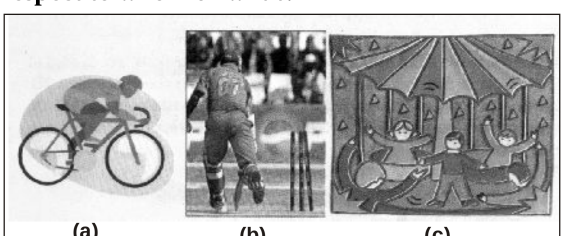


Fig.: Objects in motion

Ans. *(a)* The bicyle and cyclist are in motion with respect to the road.

(b) The Cricketer is in motion with respect to the ground, the spectators and wickets are at rest with respect to the ground.

(c) The marry go round is in motion with respect to the ground. The children on the marry go round are at rest with respect to the ground.

Q. 2. *(a)* Fill in the blanks using words *like moving, at rest, with respect to, ground, bowler*.

***(i)* A parachute falls from an aircraft. It is......with respect to the ground and......with respect to the packet attached to it.**

Ans. moving, at rest.

***(ii)* A bowler holds the ball as he runs up to bowl. The ball is at rest with respect to the..........and moving with respect to the ....**

Ans. bowler, ground.

***(iii)* Two passengers sit in two trains moving on parallel tracks in the same direction at the same speed. They are.........with respect to each other and.........with respect to the tracks.**

Ans. at rest, moving.

***(iv)* The same passengers sit in two trains moving on parallel tracks in the opposite direction at the same speed. They are..........with respect to each other and.....with respect to the tracks.**

Ans. moving, moving.

Q. 2. *(b)* Read the following sentences.

A group of people went for a boat ride on the river. Midstream, they stopped the engine of the boat and let it flow along with the current. The elderly people sat on the deck and enjoyed the scenery. The children ran around on the boat. Some young women walked around. Some men tried their hands at fishing sitting still on the deck.

Now write down, which of these objects and people are in motion, which ones are at rest, and with respect to whom or what.

Ans. Boat and people on the boat are in motion with respect to the river. Elderly people and men fishing on the boat are at rest with respect to boat.

Q. 3. *(a)* Below a cart moving in a straight line. You may use a scale to measure distances and answer the following questions:

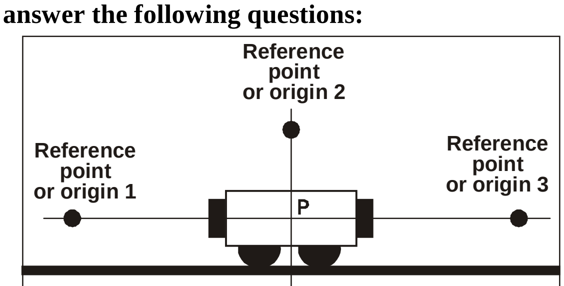


Fig.: Calculating position, distance and displacement

You may use a scale to measure distances and answer the following questions:

(i) **What is the position of the centre of the cart with respect to reference point or origin 1? Do the cart and the origin lie along a horizontal line or a vertical line? Is the position positive or negative with respect to origin 1?**

Ans. 5.0 cm, horizontal line, positive.

(ii) **What is the position of the centre of the cart with respect to reference point or origin 2? Do the cart and the origin lie along a horizontal line or a vertical line? Is the position positive or negative with respect to origin 2?**

Ans. 1.7 cm. vertical line, negative.

(iii) **What is the position of the centre of the cart with respect to reference point or origin 3? Do the cart and the origin lie along a horizontal line or a vertical line? Is the position positive or negative with respect to origin 3?**

Ans. 6.0 cm, horizontal line with negative position.

(iv) **Suppose the cart starts moving from origin 1 and travels up to the reference point 3. What is its displacement? What distance does it travel?**

Ans. 11.0 cm, towards right 11.0 cm.

(v) **Suppose the cart moves from origin 1 to point P along the horizontal path and then to the origin 2 along the vertical path. What is its displacement? What distance does it travel?**

Ans. 5.5 cm along the line pointing from origin 1 to P.

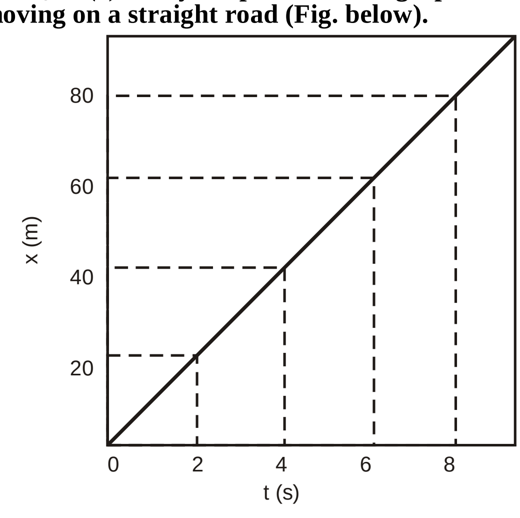
(vi) **Suppose the cart starts moving from origin 3 and travels up to the reference point 1. What is its displacement? What distance does it travel?**

Ans. 11.0 cm towards left. 11.0 cm.

(vii) **Suppose the cart moves from origin 3 to point P along the horizontal path and then to the origin 2 along the vertical path. What is its displacement? What distance does it travel?**

Ans. 6.0 cm along the line from P to origin 2.

Q. 3. ***(b)*** **Study the position-time graph of a bus moving on a straight road (Fig. below).**



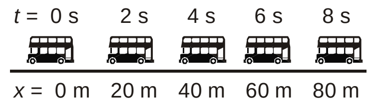


Fig. : The graph showing the positions of a moving bus at different instants of time with respect to the point $x = 0$, which is the origin.

(i) **With respect to the origin, what is the position of the bus after 2s, 3s, 5s and 7s?**

Ans.*(i)* 20 m, 30 m, 50 m, and 70 m.

(ii) **What is the distance travelled by the bus in the first 4s and from 4s to 8s?**

Ans. *(ii)* 40 m, 40 m.

(iii) **Specify its displacement with respect to the origin at 5s and between 1s and 8s.**

Ans. *(iii)* 50 m towards right, 70 m towards right.

Q. 4. ***(a)*** **Express the following vectors in unit vector notation by choosing appropriate unit vectors:**

(i) **Vector of magnitude 3 units along the positive *x*-axis.**

Ans. $3\hat{i}$, where $\hat{i}$ is the unit vector along the position *x*-axis

(ii) **Vector of magnitude 5 units along the positive y-axis.**

Ans. $5\hat{j}$, where $\hat{j}$ is the unit vector along the positive *y*-axis.

(iii) **Vector of magnitude 2 units along the positive *z*-axis.**

Ans. $2\hat{k}$, where $\hat{k}$ is the unit vector along the positive *z*-axis.

(iv) **Vector of magnitude 3 units along the northern direction.**

Ans. $3\hat{n}$, where $\hat{n}$ is the unit vector in the north direction.

(v) **Vector of magnitude 4 units along the eastern direction.**

Ans. $4\hat{e}$, where $\hat{e}$ is the unit vector in the east direction.

Q. 4. ***(b)*** **Draw each one of the following pairs of vectors and obtain their resultants:**

(i) $\vec{a} = 6\hat{i}$ **and** $\vec{b} = 8\hat{j}$

(ii) $\vec{c} = 4\hat{j}$ **and** $\vec{d} = 3\hat{i}$

(iii) $\vec{a} = 4\hat{i}$ **and** $\vec{b} = 4\hat{j}$

(iv) $\vec{b} = 2\hat{i}$ **and** $\vec{c} = 3\hat{i}$

(v) $\vec{b} = 4\hat{i}$ **and** $\vec{c} = \hat{i}$