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NUMERICAL ANALYSIS

■ ■

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QUESTION PAPER

(June – 2017)

(Solved)

NUMERICAL ANALYSIS

Time: 2 Hours] *Maximum Marks : 50*
(Weightage : 70%)

Note: *Answer* ***any five*** *questions. All computation may be done upto 3 decimal places. Use of calculators is not allowed.*

Q. 1. *(a)* Perform two iterations of the Birge-Vietta method to find a root of the polynomial

$P(x) = 2x^3 - x^2 + 2x - 2 = 0$.

Take the initial approximation $p_0 = 0.5$.

Ans. $P(x) = 2x^3 - x^2 + 2x - 2 = 0$

In this problem the coefficients are

$a_0 = -2, a_1 = 2, a_2 = -1, a_3 = 2.$

Let the initial approximation to P be $P_0 = .5$

$a_0 = -2$	$a_1 = 2$	$a_2 = -1$	$a_3 = 2$
$b_0 = 1.0$	$b_1 = 1$	$b_2 = 2.25$	$b_3 = -1.87$
$c_0 = 1.0$	$c_1 = 0.5$	$c_2 = 1.75$	

$P_1 = .826087$

$a_0 = -2$	$a_1 = 2$	$a_2 = -1$	$a_3 = 2$
$b_0 = 1.0$	$b_1 = -5.17$	$b_2 = 2.25$	$b_3 = -.44$
$c_0 = 1.0$	$c_1 = -4.35$	$c_2 = 1.50$	

$P_2 = .9676$

$a_0 = -2$	$a_1 = 2$	$a_2 = -1$	$a_3 = 2$
$b_0 = 1.0$	$b_1 = -7.25$	$b_2 = 4.25$	$b_3 = -.06$
$c_0 = 1.0$	$c_1 = -5.35$	$c_2 = 3.25$	

So one of the root of the given equation is .9676.

(b) **Solve the system of equations**

$$\begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & -2 \\ 3 & 2 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 6 \end{bmatrix}$$

using LU factorisation method.

Use $u_{11} = u_{22} = u_{33} = 1$.

Ans. [A] = [L] [U]

$$= \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$[U] = \begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & -2 \\ 3 & 2 & -4 \end{bmatrix} = \begin{bmatrix} 12 & 4 & 1 \\ 0 & 16 & -11 \\ 0 & 0 & 2 \end{bmatrix}$$

$$l_{21} = \frac{1}{4}, l_{31} = \frac{3}{4}, l_{32} = \frac{4}{2}$$

$$[L] = \begin{bmatrix} 1 & 0 & 0 \\ 1/4 & 1 & 0 \\ 3/4 & 4/2 & 1 \end{bmatrix}$$

Now from the system

$Ly = 6$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1/4 & 1 & 0 \\ 3/4 & 4/2 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 6 \end{bmatrix}$$

We get

$$y_1 = 4$$

$$\left[\frac{1}{4} + 1\right] y_2 = 4 \Rightarrow y_2 = \frac{16}{5}$$

$$\left[\frac{3}{4} + \frac{4}{2} + 1\right] y_3 = 6 \Rightarrow y_3 = \frac{24}{15}$$

and from the system $Ux = y$

$$\begin{bmatrix} 12 & 4 & 1 \\ 0 & 16 & -11 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 6 \end{bmatrix}$$

$$[12 + 4 + 1]\, x_1 = 4 \Rightarrow x_1 = \frac{4}{17}$$

$$[16 - 11]\, x_2 = 4 \Rightarrow x_2 = \frac{4}{5}$$

$$2x_3 = 6 \Rightarrow x_3 = 3.$$

Q. 2. ***(a)*** **Use Lagrange interpolation method to find the value of *y*, where *x* = 6, from the following table:**

x	1	2	7	8
y	4	5	5	4

Ans. Using Lagrange formula

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}$$

$$f(x_0) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} f(x_1) + ...+.$$

$$f(x) = \frac{(x-2)(x-7)(x-8)}{(1-2)(1-7)(1-8)} \times 4 + \frac{(x-1)(x-7)(x-8)}{(2-1)(2-7)(2-8)} \times 5 + \frac{(x-1)(x-2)(x-8)}{(7-1)(7-2)(7-8)} \times 5 + \frac{(x-1)(x-2)(x-7)}{(8-1)(8-2)(8-7)} \times 4$$

$$f(x) = \frac{(x-2)(x-7)(x-8)}{-42} \times 4 + \frac{(x-1)(x-7)(x-8)}{30} \times 5 + \frac{(x-1)(x-2)(x-8)}{-30} \times 5 + \frac{(x-1)(x-2)(x-7)}{42} \times 4$$

Put $x = 6$ then

$$f(6) = \frac{(6-2)(6-7)(6-8)}{-42} \times 4 + \frac{(6-1)(6-7)(6-8)}{30} \times 5 + \frac{(6-1)(6-2)(6-8)}{-30} \times 5 + \frac{(6-1)(6-2)(6-7)}{42} \times 4$$

$$f(6) = \frac{4 \times -1 \times -2 \times 4}{-42} + \frac{5 \times -1 \times -2 \times 5}{30} + \frac{5 \times 4 \times 2 \times 5}{30} + + \frac{5 \times 4 \times -1 \times 4}{42}$$

$$f(6) = \frac{-32}{42} + \frac{50}{30} + \frac{200}{30} - \frac{80}{42}$$

$$f(6) = \frac{-32 \times 30 + 50 \times 42 + 200 \times 42 - 80 \times 30}{42 \times 30}$$

$$f(6) = \frac{-960 + 2100 + 8400 - 2400}{1260}$$

$$f(6) = \frac{-3360 + 10500}{1260} = \frac{7140}{1260}$$

$$f(6) = 5.66.$$

(b) **Prove that:**

$$\boldsymbol{\mu\delta = \frac{1}{2}(\Delta + \nabla)}$$

Ans.
$$\mu\partial = \frac{1}{2}\left[E^{1/2} + E^{-1/2}\right]\left[E^{1/2} - E^{-1/2}\right]$$
$$= \frac{1}{2}(E - E^{-1})$$
$$= \frac{1}{2}(1 + \Delta - 1 + \nabla)$$
$$= \frac{1}{2}(\Delta + \nabla).$$

(c) **Solve the initial value problem**

$$\boldsymbol{y' = \frac{y + 2x}{y + 3x}, y(1) = 2}$$

using third order classical Runge-Kutta method. Find *y* (1.2) taking *h* = 0.2.

Ans. $y^1 = \dfrac{y + 2x}{y + 3x}, \; y(1) = 2$

Sample Preview
of
The Chapter

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NUMERICAL ANALYSIS

Solution of Non-Linear Equations in One Variable

1

Review of Calculus

INTRODUCTION

Fundamental Theorem of Calculus

If f is the derivative of F, then $\int_a^b f(x)dx = F(b) - F(a)$

Before we prove the Fundamental Theorem of Calculus, let's define a few terms.

Riemann Sum

George Friedrich Bernhard Riemann (1826-1866) was most famous for work in Non-Euclidean Geometry, differential equations, and number theory. His results in physics and mathematics form the basis of Einstein's theory of general relativity.

Let f be defined on $[a,b]$, and let be a partition of $[a,b]$ given by

$a = x_0 < x_1 < x_2 < ... < x_{n-1} < x_n = b$

where x_i is the length of the ith sub-interval. If c_i is any point in the ith sub-interval then the sum $f(c_i)\ x_i$, $x_{i-1} <= c_i <= x_i$ is called a **Riemann sum** of f for the partition.

The limit as the length of the largest sub-interval of partition (the **norm** of the partition, denoted) approaches zero (if it exists) is the definite integral, denoted.

CHAPTER AT A GLANCE

THREE FUNDAMENTALS THEOREMS

Three fundamental theorems, namely, Intermediate Value Theorem, Rolle's Theorem and Lagrange's Theorem.

Intermediate Value Theorem

If f is continuous on $[a,b]$ and k is between $f(a)$ and $f(b)$ then there must be a number, c, in $[a,b]$ such that $f(c) = k$.

The Intermediate Value Theorem can be stated in the following equivalent form:

Suppose that I is an interval in the real numbers R and that f : I –> R is a continuous function. Then the image set f (I) is also an interval.

This captures an intuitive property of continuous functions: if $f(1) = 3$ and $f(2) = 5$, then the value of f must be 4 somewhere between 1 and 2. It represents the idea that the graph of a continuous function can be drawn without lifting your pencil from the paper.

Generalization: The Intermediate Value Theorem can be seen as a consequence of the following two statements from topology:

If X and Y are topological spaces, f : X –> Y is continuous, and X is connected, then $f(X)$ is connected. A subset of R is connected if and only if it is an interval. A particular case of the Intermediate Value Theorem is this:

If f is continuous on an interval, and f is sometimes positive and sometimes negative then f must have a zero in the interval, which is known as the Weierstrass Intermediate Value Theorem, named for **Karl Weierstrass** (1815-1897), a German mathematician who is best known for his rigorous mathematical definitions of the Extreme Value Theorem and other results in calculus.

Example 1. Find the value of y in $0 \le y \le \frac{\pi}{2}$ for which $\sin(y) = \frac{1}{2}$.

Sol. We know that the function $f(y) = \sin(y) = \frac{1}{2}$ is continuous on $\left[0, \frac{\pi}{2}\right]$. Since $f(0) = 0$ and $f\left(\frac{\pi}{2}\right) = 1$, we have $f(0) < \frac{1}{2} < f\left(\frac{\pi}{2}\right)$.

Thus f satisfies all the conditions of theorem. Therefore, there are exists a point y, say y^0, such that $f(y^0) = \frac{1}{2}$.

That there exist a point y^0 such that $\sin(y^0) = \frac{1}{2}$.

Example 2. Show that the equation $2y^3 + y^2 - y + 1 = 5$ has the solution in the interval [1, 2].

Sol. Let $f(y) = 2y^3 + y^2 - y + 1$. f is the polynomial in y. f is continuous in [1, 2].

Put the $y = 1$, $f(1) = 2 \times 1^3 + 1^2 - 1 + 1 = 3$ and put the value $y = 2$, $f(2) = 2 \times 2^3 + 2^2 - 2 + 1 = 19$ and 5 lies between $f(1)$ and $f(2)$. Thus f satisfies all the condition of theorem. Therefore, there exists y_0 between 1 and 2 such that $f(y_0) = 5$. That is the equation $2y^3 + y^2 - y + 1 = 5$ has solution in the interval [1, 2].

Rolle's Theorem

Let f be continuous on $[a, b]$ and differentiable on (a, b). Then $f(a) = f(b) \Longrightarrow$ there exists a number, c, in (a, b) such that $f'(c) = 0$.

Proof: If f is a constant function, then $f'(c) = 0$ for all c in (a, b), proving this case. If $f(x) > f(a)$ for some x in (a, b) and c is a maximum of f on $[a, b]$, (c must exist, by the Extreme Value Theorem) then $f(c) \geq f(x) > f(a) = f(b)$.

Since $f(c) \neq f(a)$, and $f(c) \neq f(b)$, it follows that c is not an end-point of $[a, b]$, so it is a relative maximum.

Since relative extreme occur only at critical numbers, c is a critical number of f, which means either f is not differentiable at c or $f'(c) = 0$.

Well, f is differentiable at c, so $f'(c) = 0$, proving this case.

Similarly if $f(x) < f(a)$ for some x in (a, b) then let c be a minimum of f on $[a, b]$. This case is proved the same as above.

How is this theorem used?

A generalisation of Rolle's Theorem is the Mean Value Theorem.

Example 3. Use Rolle's theorem to show that is a solution of the equation $\cot y = y$ in $\left]0, \frac{\pi}{2}\right[$.

Sol. Here, the equation $\cot y - y = 0$. We rewrite $\cot y - y$ as $\frac{\cos y - y \sin y}{\sin y}$.

Solving the equation $\frac{\cos y - y \sin y}{\text{Sin } y} = 0$ in $\left]0, \frac{\pi}{2}\right[$ is same as solving the equation $\cos y - y \sin y = 0$.

We can find a function f which satisfies the conditions of Rolle's Theorem and for which $f(y) = \cos y - y \sin y$.

We can put the value $y = 0$, $f'(y) = 0$ and $f\left(\frac{\pi}{2}\right)$.

f satisfies all the requirements of Rolle's theorem. There a point y_0 in $[a, b]$ such that $f'(y_0) = \cos y_0 - y_0 \sin y_0 = 0$. This shows the solution equation.

Example 4. Investigate the number of roots of each of the polynomials

$P(x) = x^3 + 3x + 1$ and $q(x) = x^3 - 3x + 1$.

Sol. Since $p'(x) = 3(x^2 + 1) > 0$ for all x R, we see that p has at most one root; for if it had two (or more) roots there would be a root of $p'(x) = 0$ between them by Rolle. Since $p(0) = 1$, while $p(-1) = -3$, there is at least one root by the Intermediate Value Theorem. Hence p has exactly one root.

We have $q'(x) = 3(x^2 - 1) = 0$ when $x = \pm 1$. Since $q(-1) = 3$ and $q(1) = -1$, there is a root of q between -1 and 1 by the Intermediate Value Theorem. Looking as $x \to \infty$ and as $x \to -\infty$ shows here are three roots of q.

Example 5. Show that the equation $x - e^{-1} = 0$ has exactly one root in the interval (0, 1).

Sol. Our version of Rolle's Theorem is valuable as far as it goes, but the requirement that $f(a) = f(b)$ is sufficiently strong that it can be quite hard to apply sometimes. Fortunately the geometrical description of the result–that somewhere the tangent is parallel to the axis, does have a more general restatement.

Theorem: (The Mean Value Theorem) Let f be continuous on $[a, b]$, and differentiable on (a, b). Then there is some c with $a < c < b$ such that

$$\frac{f(b)-f(a)}{b-a} = f'(c)$$

or equivalently $f(b) = f(a) + (b - a) f'(c)$.

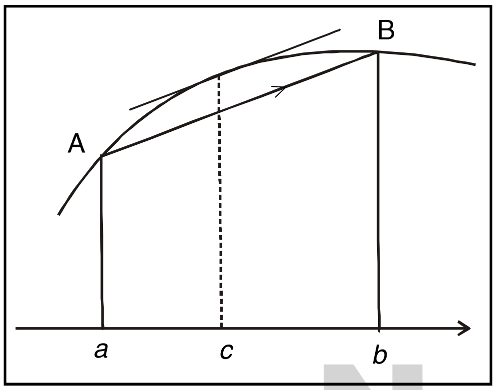

Figure: Somewhere inside a chord, the tangent to f will be parallel to the chord. The accurate statement of this common-sense observation is the Mean Value Theorem.

Proof: We apply Rolle to a suitable function; let

$$h(x) = f(b) - f(x) - \frac{f(b)-f(a)}{b-a}(b-x).$$

Then h is continuous on the interval $[a, b]$, since f is, and in the same way, it is differentiable on the open interval (a, b). Also, $f(b) = 0$ and $f(a) = 0$. We can thus apply Rolle's theorem to h to deduce there is some point c with $a < c < b$ such that $h'(c) = 0$. Thus we have

$$0 = h'(c) = -f'(c) + \frac{f(b)-f(a)}{b-a},$$

which is the required result.

Example 6. The function f satisfies $f'(x) = \frac{1}{5-x^2}$ and $f(0) = 2$. Use the Mean Value Theorem to estimate $f(1)$.

Sol. We first estimate the given derivative for values of x satisfying $0 < x < 1$. Since for such x, we have $0 < x^2 < 1$, and so $4 < 5 - x^2 < 5$. Inverting we see that

$\frac{1}{2} < f'(x) < \frac{1}{4}$ when $0 < x < 1$.

Now apply the Mean Value Theorem to f on the interval $[0, 1]$ to obtain some c with $0 < c < 1$ such that $f(1) - f(0) = f'(c)$. From the given value of $f(0)$, we see that $2.2 < f(1) < 2.25$

Example 7. The function f satisfies $f'(x) = \frac{1}{5+\sin x}$ and $f(0) = 0$. Use the Mean Value Theorem to estimate $f(\pi/2)$.

Sol. Note the 'common-sense' description of what we have done. If the derivative doesn't change much, the function will behave linearly. Note also that this gives meaning to the approximation

$$f(a + h) \approx f(a) + hf'(a).$$

We now see that the accurate version of this replaces $f'(a)$ by $f'(c)$ for some c between a and $a + h$.

Theorem: (The Cauchy Mean Value Theorem) Let f and g be both continuous on $[a, b]$ and differentiable on (a, b). Then there is some point c with $a < c < b$ such that

$$g'(c)\,(f(b) - f(a)) = f'(c)\,(g(b) - g(a)).$$

In particular, whenever the quotients make sense, we have

$$= \frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(c)}{g'(c)}$$

Proof. Let $h(x) = f(x)\,(g(b) - g(a)) - g(x)(f(b) - f(a))$, and apply Rolle's theorem exactly as we did for the Mean Value Theorem. Note first that since both f and g are continuous on $[a, b]$, and differentiable on (a, b), it follows that h has these properties. Also $h(a) = f(a)g(b) - g(a)f(b) = h(b)$. Thus we may apply Rolle to h, to deduce there is some point c with $a < c < b$ such that $h'(c) = 0$.

But $h'(c) = f'(c)(g(b) - g(a)) - g'(c)(f(b) - f(a))$

Thus $f'(c)(g(b) - g(a)) = g'(c)(f(b) - f(a))$

This is one form of the Cauchy Mean Value Theorem for f and g. If $g'(c) \neq 0$ for any possible c, then the Mean Value theorem shows that $g(b) - g(a) \neq 0$, and so we can divide the above result to get

$$= \frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(c)}{g'(c)},$$

giving a second form of the result.

Lagrange's Mean Value Theorem

Let f be continuous on $[a, b]$ and differentiable on (a, b). Then there exists a number, c, in (a, b) such that $f'(c) = (f(b) - f(a)) / (b-a)$.

Proof: Let $m = (f(b) - f(a)) / (b-a)$, the slope of the secant line that passes through points $(a, f(a))$ and $(b, f(b))$

Let $g(x) = f(x) - m(x-a)$.

Then $g(a) = f(a)$ and $g(b) = f(b) - (f(b) - f(a))(b–a)/(b–a) = f(a)$,

so $g(a) = g(b) = f(a)$

By Rolle's Theorem, there exists a number, c, in (a, b) such that $g'(c) = 0$.

$g'(x) = f'(x) - m$,

so $f'(x) = g'(x) + m$,

so $f'(c) = g'(c) + m$,

so $f'(c) = m$, proving the theorem.

How is this theorem used?

The Mean Value Theorem is used to prove the Fundamental Theorem of Calculus.

Example 8. Apply the mean value theorem to the function $f(y) = \sqrt{y}$ in [0, 2] .

Sol. The function $f(y) = \sqrt{y}$ is continuous in [0, 2] and differentiable in [0, 2].

$f(y) = \frac{1}{\sqrt[2]{y}}$. According to the theorem, there exists a point y_0 in [0, 2] such that

$$f(2) - f(0) = f'(y_0)(2 - 0)$$

Now $f(2) = \sqrt{2}$ and $f(0) = 0$ and $f(y) = \frac{1}{\sqrt[2]{y_0}}$,

therefore we have $\sqrt{2} = \frac{1}{\sqrt{y_0}}$

$$\sqrt{y_0} = \frac{1}{\sqrt{2}} \text{ and } (y_0) = \frac{1}{2}$$

The line joining end-points (0, 0) and $(2, \sqrt{2})$ of the graph of f is parallel to the tangent to the curve at the point $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$.

Example 9. Consider the function $f(y)= (y–1)(y–2)(y–3)$ in [0, 4], Find a point y_0 in [0, 4] such that $f(y) = \frac{f(4) - f(0)}{4 - 0}$

Sol. We rewrite the function $f(y)$ as

$f(y) = (y–1)(y–2)(y–3) = y^3 - 6y^2 + 11y - 6$.

Also the derivative

$f(y) = 3y - 12y - 6$

Thus f satisfies all conditions of the Mean Value Theorem. Therefore, there exists a point y_0 in such that

$$f(y_0) = \frac{f(4) - f(0)}{4 - 0}$$

i.e. $3y_0^2 - 12y_0 + 11 = \frac{6+6}{4-3} = 3$

i.e. $3y_0^2 - 12y_0 + 8 = 0$, this is quadratic equation in y_0. The root of this equation is $\frac{6+2\sqrt{3}}{8}$ and $\frac{6-2\sqrt{3}}{8}$. Taking not equal 1.732, there are two values for y_0 lying in the interval [0, 4].

Example 10. Find an approximately value $\sqrt[3]{26}$ using the mean value theorem.

Sol. Consider the function $f(y) = y^{1/3}$. Then $f(26) = \sqrt[3]{27}$.

The nearest to 26 for which the cube root is known is 27.

$f(27) = \sqrt[3]{27} = 3$. Mean Value Theorem to the function $f(y) = y^{1/3}$ in the interval [26, 27] .

The function f is continuous in [26, 27] and the derivatives is $f(y) = \frac{1}{3y^{2/3}}$, therefore , there exists a point y_0 between 26 and 27, such that

$$= \sqrt[3]{27} - \sqrt[3]{26} = \frac{1}{3y_0^{2/3}} (27–26)$$

$$= \sqrt[3]{26} = 3 - \frac{1}{3y_0^{2/3}} \quad ... (2)$$

Since y_0 is close to 27, we approximate $\frac{1}{3y_0^{2/3}}$ by $\frac{1}{3(27)^{2/3}}$, i.e. $\frac{1}{3y_0^{2/3}} \approx \frac{1}{27}$

Substituting this value in equation (2) we get

$$\sqrt[3]{26} = 3 - \frac{1}{27} = 2.963.$$

TAYLOR'S THEOREM

We have so far explored the Mean Value Theorem, which can be rewritten as

$$f(a + h) = f(a) + hf'(c)$$

where c is some point between a and $a + h$. [By writing the definition of c in this way, we have a statement that