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PROBABILITY AND STATISTICS

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Sample Preview of the Solved Sample Question Papers

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QUESTION PAPER

(June – 2017)

(Solved)

PROBABILITY AND STATISTICS

Time: 2 hours] *[Maximum Marks: 50*
(Weightage: 70%)

Note: *Question No. 7 is* ***compulsory.*** *Answer any* ***four*** *questions from questions No.* ***1*** *to* ***6.*** *Use of calculators is* ***not*** *allowed.*

Q. 1. ***(a)*** **From a bag containing 4 white and 6 red balls, three balls are drawn with replacement.**

(i) **Find the expected number of white balls drawn.**

Sol. ***(i)***

x	P(x)	x (Px)
0	$\frac{3}{5}\times\frac{7}{5}\times\frac{3}{5}=\frac{27}{125}$	0
1	$\frac{54}{125}$	$\frac{54}{125}$
2	$\frac{36}{125}$	$\frac{72}{125}$
3	$\frac{8}{125}$	$\frac{24}{125}$

Expected no. of white balls drawn = $\bar{X}$ $\sum x.P(x)$

$$\frac{\cancel{150}}{\cancel{125}} = \frac{6}{5}$$

$$= 1.2.$$

(ii) **If each white ball carries a reward of ₹ 4 and each red ball ₹ 6, find the expected reward of the draw of white balls.**

Ans. If Rs. 4 is the reward of drawing white ball, and Rs. 6 for each red ball he can win

(All the three balls are red) = 6 × 3 = 18
(1 white and 2 red balls) = 4 + 6 × 2 = 16
(2 white and 1 red ball) = 4 + 2 + 6 = 14
(All the white balls) = 4 × 3 = 12.

(b) **A computer while calculating correlation coefficient between two variables X and Y from 25 pairs of observations obtained the following results:**

$\sum$ **X = 125,** $\sum$ **X^2 = 650,** $\sum$ **Y = 100,** $\sum$ **Y^2 = 460,** $\sum$ **XY = 508.**

It was however later discovered at the time of checking that it had copied down two pairs as (6, 15) and (8, 6) while the corrected values were (8, 12) and (6, 8), respectively. Obtain the correct value of correlation coefficient.

Ans. Correct $\sum x = 125 - 6 - 8 + 8 + 6 = 125$

Correct $\sum y = 100 - 15 - 6 + 12 + 8 = 99$

Correct $\sum x^2 = 650 - 36 - 64 + 64 + 636 = 650$

Correct $\sum y^2 = 460 - 225 - 36 + 144 + 64$

$= 460 - 261 + 208$

$= 460 - 53$

$= 407$

Correct

$$\sum xy = 508 - 90 - 48 + 96 + 48$$

$$= 514$$

Correct $r = \dfrac{25\times 514 - 125\times 99}{\sqrt{25\times 650 - (125)^2}\ \sqrt{25\times 407 - (99)^2}}$

$$= \frac{12850 - 12375}{\sqrt{625}\ \sqrt{10175 - 9801}}$$

$$= \frac{475}{25\times 19} = \frac{475}{483.5} = 0.98$$

r is positive $\therefore$ y increases as x increases.

Q. 2. ***(a)*** **A random variable X has probability density function**

$f(x) = \frac{1}{2^x}$; $x = 1, 2, 3, ...$

Find its moment generating function and mean.

Ans. We know that $M_{xt} = E(e^{tx}) = \int_0^\infty e^{tx} f(n)dx$

$$E(e^{tx}) = I = \int_0^\infty e^{tx} \frac{1}{2x} dx$$

$$= \frac{e^{tx}}{t}\frac{1}{2x} - \int \frac{1}{2x}\log\frac{e^{tx}}{t}dx$$

$$I = \frac{e^{tx}}{2^{xt}} - \frac{\log 2}{t} \int \frac{1}{2x} e^{tx}\, dx$$

$$I = \frac{e^{tx}}{2^x t} - \frac{\log 2}{t} I$$

$$I + \frac{\log 2}{t} I = \frac{e^{tx}}{2^x t}$$

$$= I\left[1 + \frac{\log 2}{t}\right] = \frac{e^{tx}}{2^x t}$$

$$I = \frac{\dfrac{e^{tx}}{2^x t\left(1 + \dfrac{\log 2}{t}\right.}}{}$$

$$= \frac{e'^{\infty}}{2^x t\left(1 + \dfrac{\log 2}{t}\right)} - \frac{e'^{o}}{2^o t\left(1 + \dfrac{\log 2}{t}\right)}$$

$$= \frac{-1}{t\left[1 + \log^2\right]} = \frac{-1}{t + \log 2}$$

We can find moment about:

Origin to use $\left|\frac{d^r}{dt^r}\{Mx(t) = \left|\frac{d^r}{dt^r} M_x t\right| t = 0 = u'_r\right.$.

Where $\mu_1^1 = \left|\frac{d}{dt} - \frac{-1}{t + \log 2}\right|_{t=0}$

$$\mu_2^1 = \mu_2^1 = \left|\frac{d2}{dt^2} - \frac{-1}{t + \log 2}\right|_{t=0}$$

$$\mu_3^1 = \left|\frac{d3}{dt^3} - \frac{-1}{t + \log t}\right|_{t=0}.$$

(b) **The first three moments of a random variable X about the origin are $\frac{1}{2}(n+1)$ $\frac{1}{6}(n+1)(2n+1)$ and $\frac{1}{4}n(n+1)^2$, respectively. Find the coefficient of skewness and interpret the result.**

Ans.
$$m_1' = \frac{n+1}{2}$$

$$m_2' = \frac{(n+1)(2n+1)}{6}$$

$$m_3' = \frac{n(n+1)^2}{4}$$

$$m_2 = \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$$

$$[\because \quad m_2 = m_2' - (m_1')^2]$$

$$m_2 = \frac{n+1}{2}\left[\frac{2n+1}{3} - \frac{n+1}{2}\right]$$

$$= \frac{n+1}{2}\left[\frac{4n+2-3n-3}{6}\right]$$

$$= \frac{(n+1)(n-1)}{12}$$

$$m_2 = \frac{n^2-1}{12}$$

$$m_3 = m_3' - 3m_1'm_2' + 2m_1'^3$$

$$= \frac{n(n+1)^2}{4} - 3 \times \frac{n+1}{2} \times \frac{(n+1)(2n+1)}{6} + 2 \times \frac{(n+1)^3}{8}$$

$$= \frac{n(n+1)^2}{4} - \frac{(n+1)^2(2n+1)}{4} - \frac{(n+1)^3}{4}$$

$$= \frac{(n+1)^2}{4}[n - 2n - 1 - n - 1]$$

$$m_3 = 0$$

$$\text{Skewness} = \sqrt{\frac{m_3^2}{m_2^3}}$$

Skewness = 0 .

Q. 3. ***(a)*** **Draw the ogives for the following data and find the median from the graph:**

Class Interval	*Frequency*
5–10	6
10–15	8
15–20	17
20–25	21
25–30	15
30–35	11
35–40	2

Sample Preview of The Chapter

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PROBABILITY AND STATISTICS

DESCRIPTIVE STATISTICS

Frequency Distribution of a Character

1

INTRODUCTION

In this chapter, we will discuss the basics and terms related to statistics. Although most of the learners have been acquainted with statistics in earlier classes, but really it is necessary here. In this chapter, we shall discuss the methods of how to collect the data as well as to organise these data (i.e. concept of frequency distribution). We will also study the various models of frequency distribution not only in tabular form, but also in diagrammatic representation. Thus, our objective of this chapter study is:

- To define qualitative and quantitative character and differences between the two.
- To define a discrete and a continuous variable and differences between the two.
- To draw the frequency table with their relative frequencies, cumulative frequencies and frequency densities.
- To explain the diagrammatical presentation of various frequency distributions.

CHAPTER AT A GLANCE

RAW MATERIALS OF STATISTICS

In our daily life we have seen the various terms related to statistics, so we have to learn firstly, the meaning of statistics. Statistics is defined in two defferent contexts: **numerical data** and **discipline.** We will understand it by some examples like the statistics of run sconed by the **Chennai Superkings** in IPL-2 matches, statistics of marks obtained by the students in Economics in an examination, etc., these are termed as numerical data.

On the contrary, a player of the Chennai Superkings or a student of Economics are called the discipline. Thus, we can say that the numerical data arises in the ambid of life whereas the disciplines itself relates to the collection, analysis and interpretation of data. Hence the combination of numerical data and disciplines is known as **Statistics.**

Two Basic Concepts Regarding the Statistics

Let us consider the two basic concepts regarding to the statistical study: One is **character** and another is **individual.** Again we will understand these two concepts through an example:

- Suppose a teacher has awarded to his students on their performance in an examination by grades (A, B, C, D and E). Here, students are individuals and the grades are the characters.

Thus, through this example, we can say that one attribute who gets the benefits is called **individual** and types of attributes/benefits are called **characters.**

- **Sources of Data:** In any study/research, we collect the data from two sources: **Primary** and **Secondary.**

***(i)* Primary Data Sources:** In any study/research, when we collect the data on the relevant groups of individuals by survey method, it is called the **primary source.**

***(ii)* Secondary Data Sources:** In any study/research when we take data from which is already published by the Government or any other agency, is called **secondary source.**

Note: In using the secondary data sources, we shall be careful that data should be reliable and relevant to their study.

- **Methods of Collecting Primary Data**

There are various methods to collect the primary data, one of the most popular method is direct observation, which is as under:

Direct Observation: Through counting or measurement or by inspection, when we collect the required information, then this type of observation is called **direct observation**.

One who provides the information is called informant.

In direct observation we collect the information directly by informants or through enumerators by following methods:

(1) Questionnaire Method: In this method, the enumerators collect the data by filling out the questionnaire forms. This method is very useful for the educated informants.

(2) Interview Method: This method is generally useful for illiterate or uneducated informants. In this method, enumerators collect data/schedule by a thorough and logical questioning of each informant.

- **Classification of Characters**

We have discussed earlier that character is the types of attributes/benefits, which are classified into two broad categories: one is qualitative and another is quantitative character.

(1) Qualitative Character: Such a character that can't be counted or expressed numerically, but it has various forms for different individuals are called qualitative characters. As for example, the brand name of the motorbikes in Delhi is a character: it may be Herohonda, Yamaha, Bajaj etc., whose possible forms can be differentiated orally but not numerically, is called qualitative character.

(2) Quantitative Character: Such a character that can be counted or expressed numerically for different individuals is called quantitative character. As for example, when we would ask a question how many motorbikes of Herohonda in Delhi, it can be counted and such type of character is called quantitative character.

- **Distinction between Qualitative Character and Quantitative Character**

Qualitative Character	Quantitative Character
● It can't be expressed numerically, but observed orally.	● It is expressed only numerically.
● It is generally known as attributes.	● It is known as variable.
● It is observable through the ranking of the preferences.	● It has no need for making the rank.

Now, let us consider the quantitative character which is classified into two parts: one is **discrete** and another is **continuous variable.**

***(i)* Discrete Variable:** It is the variable which is not observable in internal, but which can be conceivable only some actual or isolated variables. As for example, the size of the family takes values like 1, 2, 3..... etc., height of the children like. 2 ft, 4 ft, 3.5 ft.....etc. are called discrete variables.

***(ii)* Continuous Variable:** It can take any values in some interval, say the ages of the teachers between 25 years to 55 years, number of students of primary school between 5 years to 12 years, etc., are called the continuous variable. Similarly, suppose the lower level of an interval is 'β' and upper level of that interval is 'α', then the continuous variable is defined as $[\alpha, \beta]$ of the given data.

FREQUENCY DISTRIBUTIONS

Earlier, we have studied more about the collection of data. Now we will be acquainted with the organisation of data through frequency distribution. For the comfortable study of the frequency distribution, we, therefore, categorise the frequency distribution into two parts: ungrouped frequency distribution and grouped frequency distribution.

Now, let us start with the ungrouped frequency distribution.

Ungrouped Frequency Distribution: Ungrouped frequency distribution might have the data with qualitative in nature or the variable with discrete. So, we shall first discuss the ungrouped frequency distribution with qualitative character and then the ungrouped frequency distribution with discrete variable.

(1) Ungrouped Frequency Distribution of Qualitative Character: This concept can be easily understood by an illustration. Let us consider a college conducts a graduation examination which consists of four subjects like, Statistics, Economics, Mathematics and English. There are 100 students have passed in all four subjects, which is shown in a frequency distribution table as:

Table 1: Frequency Distribution of the Passed Students

Subjects	No. of Students	Relative Frequency
Statistics	30	30/100 = 0.3
Economics	20	20/100 = 0.2
Mathematics	40	40/100 = 0.4
English	10	10/100 = 0.1
Total	100	1.0

Table 1 shows the frequency distribution of 100 students who have passed in four subjects like Statistics, Economics, Mathematics and English.

- The data of the second column are called frequencies of the four subjects.
- Column 1 and 2 show the frequency distribution among 100 students in four subjects.
- Column 3 shows the relative frequency of that four subjects by this formula:

$$\text{Relative frequency of a subject} = \frac{\text{Frequency of that subject}}{\text{Total frequency}}$$

As for example:

Relative frequency of statistics

$$= \frac{30}{100} = 0.3$$

Similary we can find all relative frequencies, which are shown in Table 1.

Note:
- A frequency must be non-negative.
- A relative frequency must be a rational number in the interval [0, 1].

If qualitative character is classified just in two classes known as Dichotomy.

(2) Ungrouped Frequency Distribution of a Discrete Variable: This concept is also to be understand through an illustration. Let us consider an economist collects the data on household size from so households of rural locality, which is shown in Table-2.

Table 2: Data of Houshold Size of 80 Rural Households

8	4	4	3	7	8	3
3	2	4	9	6	1	2
5	3	5	4	5	7	1
5	2	4	4	5	4	4
3	4	5	5	6	5	5
4	4	2	4	5	2	5
4	3	5	5	6	6	6
5	3	7	2	7	6	2
8	1	6	5	6	6	9
7	9	5	4	5	5	3

Now we create the frequency table of these discrete values of the households, which is shown in Table 3.

Table 3: Frequency Distribution for the Households Size of 80 Rural Households

Household Size	Tally Marks	Frequency	Relative Frequency
1	𝍸	3	3/80 = 0.0375
2	𝍸 |||	8	8/10 = 0.1000
3	𝍸 𝍸	10	10/80 = 0.1250
4	𝍸 𝍸 𝍸	15	15/80 = 0.1875
5	𝍸 𝍸 𝍸 𝍸	20	20/80 = 0.2500
6	𝍸 𝍸 |	11	11/80 = 0.1375
7	𝍸 ||	6	6/80 = 0.0750
8	||||	4	4/80 = 0.0500
9	|||	3	3/80 = 0.0375
Total		80	1.0000

- **Cumulative Frequency Distribution for the Discrete Variable:** There are two other ways to represent the frequency distribution of the discrete variable. These are: **Less than type** and **More than type** cumulative frequency distribution.

Table 4: Cumulative frequency distribution of the 'less than type' and 'more than type' of household size of 80 rural households

Less than type of cumulative frequency distribution			More than type of cumulative frequency distribution		
Household size	**Frequency**	**Cumulative frequency**	**Household size**	**Frequency**	**Cumulaties frequency**
Less than 1	3	3	More than 1	3	80
Less than 2	8	11	More than 2	8	77
Less than 3	10	21	More than 3	10	69
Less than 4	15	36	More than 4	15	59
Less than 5	20	56	More than 5	20	44
Less than 6	11	67	More than 6	11	24
Less than 7	6	73	More than 7	6	13
Less than 8	4	77	More than 8	4	7
Less than 9	3	80	More than 9	3	3
	80		Any value more than 9		0

Procedure to Create Less Than Type of Cumulative Frequency: If we have to find the less than type of cumulative frequency of a particular row, we add the previous frequency with its frequency/ cumulative frequency. Like as, we have to find the cumulative frequency we add as, 3 + 8 = 11, similarly for the 3rd row, we add 11 + 10 = 21 and so on.

Note: Cumulative frequency of the first row is the same as the frequency of that row and cumulative frequency of the last row equals to the total frequency.

Procedure to Create More Than Type of Cumulative Frequency: If we have to find the more than type of cumulative frequency of a particular row, we substract the total frequency/remaining cumulative frequency the previous frequency. Like as, when we have to find the more than type of cumulative frequency of 2nd row, we subtract as 80 – 3 = 77, similarly for 3rd row, we subtract 77 – 8 = 69 ..... and so on.

Note: Cumulative frequency of the first row is the total frequency (i.e. 80) and last row is zero.

Relative cumulative frequency table is also made by the same procedure as relative frequency. But formula is $\frac{\text{Cumulative frequency}}{\text{Total frequency}}$.

Grouped Frequency Distribution: Till now we have learned about the ungrouped frequency table for both qualitative as well as discrete variable. But ungrouped frequency distribution has some limitations, it cannot construct the frequency table for continuous variable. Continuous variable have infinitely many distinct values. Thus it is necessary to group the some variables together and then construct a frequency table. This is called the 'grouped frequency distribution'.

Let us consider the following table, we cannot make easily the ungrouped frequency distribution:

Table 5: Height of 80 children below the 15 years

(in ft.)

4.5	6.1	2.5	5.1	1.9	2.1	2.9	3.5
2.6	3.2	3.5	2.5	2.9	2.2	3.3	3.3
5.5	4.2	2.3	6.2	3.3	3.2	3.5	2.3
6.3	2.6	4.3	2.3	2.3	4.3	3.6	5.2
4.0	3.5	4.1	3.2	5.6	5.0	4.2	4.2
5.5	1.3	5.1	3.5	2.4	2.5	4.3	2.5
3.6	2.6	3.3	4.2	5.1	3.6	5.1	6.1
2.6	3.5	3.5	4.6	4.9	6.1	2.6	3.4
4.3	4.3	5.0	2.1	4.5	2.1	5.6	4.8
2.3	6.2	1.8	2.5	4.4	3.4	3.2	3.8

This table shows the heights of the 80 children belonging to a locality, whose ages are under 15 years. Now, we shall construct a frequency table from above data as follows: