

NEERAJ[®]

LINEAR PROGRAMMING

By:

Nirman Kaur, M.C.A.

Reference Book
Including
Solved Question Papers

New Edition



NEERAJ PUBLICATIONS

(Publishers of Educational Books)

(An ISO 9001 : 2008 Certified Company)

1507, 1st Floor, NAI SARA, DELHI - 110006

Ph.: 011-23260329, 45704411, 23244362, 23285501

E-mail: info@neerajbooks.com

Website: www.neerajbooks.com

Price
₹ 200/-

Published by:

NEERAJ PUBLICATIONS

Sales Office : 1507, 1st Floor, Nai Sarak, Delhi-110 006

E-mail: info@neerajbooks.com

Website: www.neerajbooks.com

Reprint Edition with Updation of Sample Question Paper Only

Typesetting by: *Competent Computers*

Printed at: *Novelty Printer*

Notes:

1. For the best & up-to-date study & results, please prefer the recommended textbooks/study material only.
2. This book is just a Guide Book/Reference Book published by NEERAJ PUBLICATIONS based on the suggested syllabus by a particular Board/University.
3. The information and data etc. given in this Book are from the best of the data arranged by the Author, but for the complete and up-to-date information and data etc. see the Govt. of India Publications/textbooks recommended by the Board/University.
4. Publisher is not responsible for any omission or error though every care has been taken while preparing, printing, composing and proof reading of the Book. As all the Composing, Printing, Publishing and Proof Reading, etc. are done by Human only and chances of Human Error could not be denied. If any reader is not satisfied, then he is requested not to buy this book.
5. In case of any dispute whatsoever the maximum anybody can claim against NEERAJ PUBLICATIONS is just for the price of the Book.
6. If anyone finds any mistake or error in this Book, he is requested to inform the Publisher, so that the same could be rectified and he would be provided the rectified Book free of cost.
7. The number of questions in NEERAJ study materials are indicative of general scope and design of the question paper.
8. Question Paper and their answers given in this Book provide you just the approximate pattern of the actual paper and is prepared based on the memory only. However, the actual Question Paper might somewhat vary in its contents, distribution of marks and their level of difficulty.
9. Any type of ONLINE Sale/Resale of "NEERAJ IGNOU BOOKS/NEERAJ BOOKS" published by "NEERAJ PUBLICATIONS" on Websites, Web Portals, Online Shopping Sites, like Amazon, Flipkart, Ebay, Snapdeal, etc. is strictly not permitted without prior written permission from NEERAJ PUBLICATIONS. Any such online sale activity by an Individual, Company, Dealer, Bookseller, Book Trader or Distributor will be termed as ILLEGAL SALE of NEERAJ IGNOU BOOKS/NEERAJ BOOKS and will invite legal action against the offenders.
10. Subject to Delhi Jurisdiction only.

© Reserved with the Publishers only.

Spl. Note: This book or part thereof cannot be translated or reproduced in any form (except for review or criticism) without the written permission of the publishers.

Get Books by Post (Pay Cash on Delivery)

If you want to Buy NEERAJ BOOKS for IGNOU Courses then please order your complete requirement at our Website www.neerajbooks.com . where you can select your Required NEERAJ IGNOU BOOKS after seeing the Details of the Course, Name of the Book, Printed Price & the Cover-pages (Title) of NEERAJ IGNOU BOOKS.

While placing your Order at our Website www.neerajbooks.com You may also avail the Various "Special Discount Schemes" being offered by our Company at our Official website www.neerajbooks.com.

We also have "Cash of Delivery" facility where there is No Need To Pay In Advance, the Books Shall be Sent to you Through "Cash on Delivery" service (All The Payment including the Price of the Book & the Postal Charges etc.) are to be Paid to the Delivery Person at the time when You take the Delivery of the Books & they shall Pass the Value of the Goods to us. We usually dispatch the books Nearly within 3-4 days after we receive your order and it takes Nearly 4-5 days in the postal service to reach your Destination (In total it take nearly 8-9 days).



NEERAJ PUBLICATIONS

(Publishers of Educational Books)

(An ISO 9001 : 2008 Certified Company)

1507, 1st Floor, NAI SARAK, DELHI - 110006

Ph. 011-23260329, 45704411, 23244362, 23285501

E-mail: info@neerajbooks.com Website: www.neerajbooks.com

CONTENTS

LINEAR PROGRAMMING

<i>Question Paper—June, 2017 (Solved)</i>	1-11
<i>Question Paper—June, 2016 (Solved)</i>	1-10
<i>Question Paper—June, 2015 (Solved)</i>	1-4
<i>Question Paper—June, 2014 (Solved)</i>	1-5
<i>Question Paper—June, 2013 (Solved)</i>	1-6

<i>S.No.</i>	<i>Chapter</i>	<i>Page</i>
--------------	----------------	-------------

BASIC MATHEMATICS AND OPTIMIZATION

1.	Basic Algebra	1
2.	Inequalities and Convex Sets	16
3.	Optimization in Two Variables	28
4.	Optimization in More than Two Variables	38
	Review Exercises–I	43

SIMPLEX METHOD AND DUALITY

5.	Standard Form and Solutions	49
6.	Simplex Method	62
7.	Primal and Dual	78
8.	Duality Theorems	84
	Review Exercises–II	88

**Sample Preview
of the
Solved
Sample Question
Papers**

Published by:



**NEERAJ
PUBLICATIONS**

www.neerajbooks.com

QUESTION PAPER

(June - 2017)

(Solved)

LINER PROGRAMMING

Time: 2 hours]

[Maximum Marks: 50

Note: Question No. 1 is compulsory. Attempt any four questions out of questions no. 2 to 7. Use of calculators is not allowed.

Q. 1. State which of the following statements are true and which are false. Give reasons for your answer with a short proof or a counter-example.

(a) The LPP, maximizing $z = -5x_2$ subject to $x_1 + x_2 \leq 1$, $0.5x_1 + 5x_2 \geq 0$ and $x_1 \geq 0$, $x_2 \geq 0$, has no feasible solution.

Ans. The LPP Max $z = -5x_2$
Subject to constraints

$$x_1 + x_2 \leq 1$$

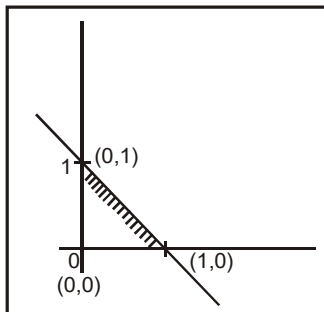
$$0.5x_1 + 5x_2 \geq 0$$

and $x_1 \geq 0, x_2 \geq 0$

Has no feasible solution.

Statement is true

Reason because the constraints are plotted on graph as shown in fig. Since there are no unique feasible solution space, therefore a unique set of values of variables x_1 and x_2 that satisfy all the constraints cannot be determine. Hence, there is no feasible solution to this LP problem of the conflicting constraints.



(b) If a negative value appears in the solution column (X_B) of the simplex method, then the basic solution is unbounded.

Ans. If a negative value appears in the solution column (X_B) of the simplex table than the basic

solution is unbounded (True). Because current solution is not optimal solution. Column corresponding to this is key column and decision variable is incoming variable. But all minimum ratio (X_B) are negative. That means decision variable can be increased indefinitely without violating a constraint. Solution is therefore unbounded one.

(c) If the variables in a primal problem are all greater than, or equal to zero, then the variables in the dual problem would be less than or equal to zero.

Sol. Statement is false

Reason-Ex-Primal LP Problem

$$\text{Max } Zx = \sum_{j=1}^n c_j \times x_j$$

Subject to the constraints

$$\sum_{j=1}^n a_{ji} x_j \leq b_i \text{ (if the variable in a primal problem$$

are all less than or equal to zero)

$$\text{and } x_j \geq 0, \text{ for all } j$$

Dual LP Problem

$$\text{Min } zy = \sum_{i=1}^m b_i y_i$$

Subject to the constraints

$$\sum_{j=1}^n a_{ji} y_j \geq c_j \text{ (if the variable in the dual problem$$

would be greater than or equal to zero)

$$\text{and } y_i \geq 0, \text{ for all } i$$

(d) The value of the game

	I	II	III	IV
I	-5	3	1	5
II	5	5	0	6
III	-4	-2	4	5
IV	7	2	6	8

is 6.

Ans. Value of game is 6 (False)

		Player B				
		I	II	III	IV	Row minimum
Player A	I	-5	3	1	5	1
	II	5	5	0	6	0 maximum
	III	-4	-2	4	5	-4
	IV	7	2	6	8	2

Column maximum 7 [5] 6 8
 ← minimax

[Maximum ≠ Minimax]

Game has no saddle point that means mixed strategy games (without saddle point). So value of game is not equal to 6.

(e) The minimum number of lines covering all zeros in a reduced cost matrix of order n in an assignment problem can be at least n .

Ans. The minimum number of line covering all zeros in a reduced cost matrix of order (n) in an assignment problem can be atleast (n). Statement are true because assignment problem solve exact square matrix ($n \times n$). So atleast n no. of lines cover all zeros for (n) no. of allocation.

Q. 2. (a) Formulate the dual of the following LPP:

Maximize $z = 2x_1 + x_2$

Subject to the constraints

$x_1 + 5x_2 \leq 10$

$x_1 + 3x_2 \geq 6$

$2x_1 + 2x_2 \leq 8$

$x_2 \geq 0$ and x_1 unrestricted.

The dual must have exactly two constraints and three variables.

Sol. Formulate the dual of the following L.P.P.

Maximize $Z = 2x_1 + x_2$

Subject to the constraints

$x_1 + 5x_2 \leq 10$

$x_1 + 3x_2 \geq 6$
 $\Rightarrow -x_1 - 3x_2 \leq -6$
 $2x_1 + 2x_2 \leq 8$

$x_2 \geq 0$ and x_1 unrestricted.

Minimize $y = 10y_1 - 6y_2 + 8y_3$

Subject to the constraints

$y_1 - y_2 + 2y_3 \geq 2$

$5y_1 - 3y_2 + 2y_3 \geq 1$

$y_2 \geq 0, y_3 \geq 0$

and y_1 is unrestricted.

(b) Solve graphically the following LPP:

Maximize $z = 3x_1 + 2x_2$

Subject to the constraints

$-2x_1 + x_2 \leq 1$

$x_1 \leq 2$

$x_1 + x_2 \leq 3$

$x_1, x_2 \geq 0$.

Ans. Maximize $z = 3x_1 + 2x_2$

Subject to the constraints.

$2x_1 + x_2 \leq 1$

$x_1 \leq 2$

$x_1 + x_2 \leq 3$

$x_1, x_2 \geq 0$

First, write the inequality of the constraints into an equation and plot the line in the graph.

$2x_1 + x_2 = 1$ passes through (0, 1) (1/2, 0)

$x_1 = 2$ passes through (2, 0)

$x_1 + x_2 = 3$ passes through (0, 3) (3, 0)

We marks the region below the lines lying in the first quadrant as the inequality of the constraints are \leq .

The feasible region is ABCDE

Corner points	Values of $z = 3x_1 + 2x_2$
A (1/2, 0)	Max $Z_A = 3/2 = 1.5$
B (0, 1)	$Z_B = 2$
C (0, 3)	$Z_C = 6$
D (2, 1)	$Z_D = 8$
E (3, 0)	$Z_E = 6$

Sample Preview of The Chapter

Published by:



**NEERAJ
PUBLICATIONS**

www.neerajbooks.com

LINEAR PROGRAMMING

Basic Mathematics and Optimization

Basic Algebra

1

INTRODUCTION

Algebra is the branch of mathematics that uses letters in place of some unknown numbers. During your early schooling, you must have learnt formulas in algebraic equation like area of a rectangle: $A = w \times h$. The letters denote some unknown numbers, once we knew the value of width and height; we could substitute them into the formula. Thus, algebra is the study of the properties of operations on numbers. It is a problem solving tool. The linear equation is an algebraic equation in which each term is either a constant or the product of a constant and a single variable. Linear equations can have one or more variables. Various elementary methods are used to solve these linear equations which involve two or three variables. But these methods are not helpful where a large number of equations involve more than three variables. Thus, few other methods are involved for such equations; such as Matrices and Determinants. By the end of this unit, you will be able to define a matrix and a determinant, addition and multiplication of two matrix, obtain the

determinant of a matrix, and compute the inverse of a matrix, Matrices and Determinants.

CHAPTER AT A GLANCE

MATRICES AND DETERMINANTS

Matrix: A matrix is an ordered rectangular array of numbers. The numbers or entries are called the elements of the matrix. The rectangular arrays of entries are enclosed in an ordinary bracket or in a square bracket. They can be used to represent systems of linear equations. Consider the following arrangement:

$$\begin{bmatrix} 3 & 4 & 2 \\ 6 & 8 & 9 \end{bmatrix}$$

In this arrangement, there are two rows and three columns. The number 8 lies in the 2nd row and 2nd column.

$$A = \begin{bmatrix} 4 & 7 \\ 9 & 5 \\ 3 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 10 & 4 \\ 14 & 17 \end{bmatrix}$$

2 / NEERAJ : LINEAR PROGRAMMING

In the above arrangement, matrix A has 3 rows and 2 columns and the order is 3×2 . Matrix B has 2 rows and 2 columns and the order is 2×2 . Capital letters are used to denote the matrix. The system of linear equations can also be converted into matrix:

$$3x - 5y + z = 6$$

$$4x + y - z = 8$$

$$x + y - z = 1$$

$$A = \begin{bmatrix} 3 & -5 & 1 \\ 4 & 1 & -1 \\ 1 & 1 & -1 \end{bmatrix}$$

$$A = \begin{bmatrix} -5 & 1 & 8 \\ 1 & -6 & -2 \\ 9 & 4 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} -5 & 1 & 8 \\ 1 & -6 & -2 \\ 9 & 4 & 2 \end{bmatrix}$$

Are these matrices equal? Yes, because

- Both the matrices have the same dimensions i.e. 3 rows and 3 columns (3×3)
- All the corresponding elements are same in both matrices.

Example: Which of the following matrices are equal?

Definition of a Matrix

- Rectangular array of numbers.
- A matrix with m rows and n columns is called a matrix of order $m \times n$.
- First subscript is row and second subscript is column.
- A row matrix is a matrix having only one row.
- A column matrix is a matrix having one column.
- If the number of rows and columns are same in a matrix, it is known as a square matrix.

$$A = \begin{bmatrix} 3 & 1 & 18 \\ 1 & -2 & -12 \\ 5 & 8 & -6 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & 1 & 18 \\ 1 & -4 & -12 \\ 5 & 8 & 6 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & 1 & 18 \\ 1 & -2 & -12 \\ 5 & 8 & -6 \end{bmatrix}$$

$$A^{m \times n} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \cdot & \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix}$$

Matrix A and C are equal. Since they have same dimensions and equal corresponding elements.

Square Matrix

A square matrix is a matrix which has the same number of rows and columns. Consider the following matrix:

Equal Matrix

Two matrices can be equal if and only if these matrices have the same dimensions and equal corresponding elements. Let's consider following two matrices; named A and B:

$$A = \begin{bmatrix} -2 & -8 & 4 \\ 4 & 9 & -13 \\ 3 & 1 & 6 \end{bmatrix}$$

It is a square matrix, as it is having 3 rows and 3 columns (3×3).

Let us consider following matrix:

$$A = \begin{bmatrix} 0 \\ 4 \\ 17 \end{bmatrix}$$

$$B = [0 \ 4 \ 1 \ 1]$$

The A matrix is in order of 3×1 (3 rows and 1 column) and B matrix is in order of 1×4 (1 row and 4 columns). **When a matrix is having only one column is called a column matrix**, thus A is a column matrix and **when a matrix is having only one row is called a row matrix**, thus B is a row matrix. Consider the following matrix:

$$A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

It is a square matrix having all elements as zero except the diagonal elements. Such types of matrix are known as **Diagonal Matrices**. The diagonal elements are: 4, 9, and 5.

Identity Matrix: An Identity matrix is a diagonal matrix with all diagonal element = 1.

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Zero Matrix: When all the elements of a matrix are 0, then that matrix is called a Zero matrix or Null matrix. For example.

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= B \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C = [0 \ 0 \ 0 \ 0]$$

Submatrix

A sub matrix is formed by selecting certain rows and columns from a bigger matrix or if we delete few rows and columns from a matrix, the resulting matrix is called a sub matrix.

$$A = \begin{bmatrix} 2 & 8 & 18 & 4 \\ 9 & -2 & 13 & 3 \\ 5 & 6 & 9 & 1 \\ 1 & 3 & 7 & 9 \end{bmatrix}$$

If we delete third row and second column, we obtain the following matrix:

$$B = \begin{bmatrix} 2 & 18 & 4 \\ 9 & 13 & 3 \\ 1 & 7 & 9 \end{bmatrix}$$

Scalar Multiplication

In scalar multiplication, each element of the matrix is multiplied by the number or scalar. For example, if

$$A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \\ 6 & 7 \end{bmatrix}, \text{ determine } 3A.$$

Solution:

$$A = \begin{bmatrix} 2 \times 3 & 4 \times 3 \\ 1 \times 3 & 3 \times 3 \\ 6 \times 3 & 7 \times 3 \end{bmatrix} = \begin{bmatrix} 6 & 12 \\ 3 & 9 \\ 18 & 21 \end{bmatrix}$$

Addition of Matrix

If A and B are two matrix of the same size. Then their sum $C = A + B$ is defined by the following rule:

$$C_{ij} = A_{ij} + B_{ij}$$

Example:

$$A = \begin{bmatrix} 3 & 5 & 9 \\ 5 & 1 & 3 \\ -2 & 4 & 0 \end{bmatrix}$$

4 / NEERAJ : LINEAR PROGRAMMING

$$B = \begin{bmatrix} 2 & 1 & 0 \\ 3 & 6 & -4 \\ 5 & 3 & 1 \end{bmatrix}$$

Find A + B.

Solution:

$$A = \begin{bmatrix} 3+2 & 5+1 & 9+0 \\ 5+3 & 1+6 & 3+(-4) \\ -2+5 & 4+3 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 6 & 9 \\ 8 & 7 & -1 \\ 3 & 7 & 1 \end{bmatrix}$$

Difference of Matrix

If A and B are two matrix of the same size. Then their difference $C = A - B$ is defined by the following rule:

$$C_{ij} = A_{ij} - B_{ij}$$

Example:

$$A = \begin{bmatrix} 4 & 3 & 1 \\ 6 & -2 & 9 \end{bmatrix}$$

$$B = \begin{bmatrix} -3 & 2 & 0 \\ 1 & 3 & 5 \end{bmatrix}$$

Find A - B.

Solution:

$$A = \begin{bmatrix} 4-(-3) & 3-2 & 1-0 \\ 6-1 & -2-3 & 9-5 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 1 & 1 \\ 5 & -5 & 4 \end{bmatrix}$$

Product of Matrix

When the number of columns of the first matrix is the same as the number of rows in the second matrix then product of matrix can be performed.

Matrix multiplication for 2×2 matrix:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$$

Matrix multiplication for 3×3 matrix:

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} j & k & l \\ m & n & o \\ p & q & r \end{bmatrix}$$

$$= \begin{bmatrix} aj+bm+cp & ak+bn+cq & al+bo+cr \\ dj+em+fp & dk+en+fq & dl+eo+fr \\ gj+hm+ip & gk+hn+iq & gl+ho+ir \end{bmatrix}$$

Example:

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 5 & 4 \\ 6 & 8 & 7 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 5 & 8 \\ 4 & 3 & 2 \end{bmatrix}$$

Find the product of two matrix i.e. $A \times B$.

Sol.

$$\begin{bmatrix} 2 & 3 & 1 \\ 3 & 5 & 4 \\ 6 & 8 & 7 \end{bmatrix} \times \begin{bmatrix} 3 & 2 & 6 \\ 1 & 5 & 8 \\ 4 & 3 & 2 \end{bmatrix}$$