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LINEAR PROGRAMMING

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CONTENTS

LINEAR PROGRAMMING

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Sample Question
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QUESTION PAPER

(June – 2017)

(Solved)

LINEAR PROGRAMMING

Time: 2 hours] *[Maximum Marks: 50*

Note: *Question No. **1** is **compulsory.** Attempt any **four** questions out of questions no. **2** to 7. Use of calculators is **not** allowed.*

Q. 1. State which of the following statements are *true* and which are *false.* Give reasons for your answer with a short proof or a counter-example.

(a) **The LPP, maximizing $z = -5x_2$ subject to $x_1 + x_2 \leq 1$, $0.5x_1 + 5x_2 \geq 0$ and $x_1 \geq 0$, $x_2 \geq 0$, has no feasible solution.**

Ans. The LPP Max $z = -5x_2$

Subject to constraints

$$x_1 + x_2 \leq 1$$

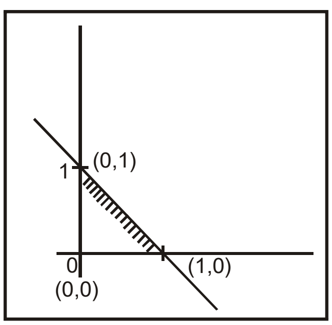
$$0.5\, x_1 + 5x_2 \geq 0$$

and $x_1 \geq 0, x_2 \geq 0$

Has no feasible solution.

Statement is true

Reason because the constraints are plotted on graph as shown in fig. Since there are no unique feasible solution space, therefore a unique set of values of variables x_1 and x_2 that safisfy all the constraints cannot be determine. Hence, there is no feasible solution to this LP problem of the conflicting constraints.



(b) **If a negative value appears in the solution column (X_B) of the simplex method, then the basic solution is unbounded.**

Ans. If a negative value appears in the solution column (X_B) of the simplex table than the basic solution is unbounded **(True)**. Because current solution is not optimal solution. Column corresponding to this is key column and decision variable is incoming variable. But all minimum ratio (X_B) are negative. That means decision variable can be increased indefinitely without violating a constraint. Solution is therefore unbounded one.

(c) **If the variables in a primal problem are all greater than, or equal to zero, then the variables in the dual problem would be less than or equal to zero.**

Sol. Statement is false

Reason–E*x*–Primal LP Problem

$$\text{Max } Zx = \sum_{j=1}^{n} c_i \times j$$

Subject to the constraints

$$\sum_{j=1}^{n} a_{ji}\, x_j \leq b_i$$ (if the variable in a primal problem are all less than or equal to zero)

and $x_j \geq$, for all j

Dual LP Problem

$$\text{Min } zy = \sum_{i=1}^{m} b_i\ y_i$$

Subject to the constraints

$$\sum_{j=1}^{n} a_{ji}\ y_j \geq c_j$$ (if the variable in the dual problem would be greater than or equal to zero)

and $y_i \geq 0$, for all i

***(d)* The value of the game**

$$\begin{array}{c} \\ \textbf{I} \\ \textbf{II} \\ \textbf{III} \\ \textbf{IV} \end{array} \begin{array}{c} \begin{array}{cccc} \textbf{I} & \textbf{II} & \textbf{III} & \textbf{IV} \end{array} \\ \begin{bmatrix} -5 & 3 & 1 & 5 \\ 5 & 5 & 0 & 6 \\ -4 & -2 & 4 & 5 \\ 7 & 2 & 6 & 8 \end{bmatrix} \end{array}$$

is 6.

Ans. Value of game is 6 **(False)**

Player B

$$\text{Player A} \begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \end{array} \begin{array}{c} \begin{array}{cccc} \text{I} & \text{II} & \text{III} & \text{IV} \end{array} \\ \begin{bmatrix} -5 & 3 & 1 & 5 \\ 5 & 5 & 0 & 6 \\ -4 & -2 & 4 & 5 \\ 7 & 2 & 6 & 8 \end{bmatrix} \end{array} \begin{array}{c} \text{Row minimum} \\ 1 \\ 0 \text{ maximum} \\ -4 \\ 2 \end{array}$$

Column maximum 7 $\boxed{5}$ 6 8
minimax

[Maximum $\neq$ Minimax]

Game has no saddle point that means mixed strategy games (without saddle point). So value of game is not equal to 6.

***(e)* The minimum number of lines covering all zeros in a reduced cost matrix of order *n* in an assignment problem can be at least *n*.**

Ans. The minimum number of line covering all zeros in a reduced cost matrix of order (n) in an assignment problem can be atleast (n). Statement are true because assignment problem solve exact square matrix ($n \times n$). So atleast n no. of lines cover all zeros for (n) no. of allocation.

Q. 2. *(a)* Formulate the dual of the following LPP:

Maximize $z = 2x_1 + x_2$

Subject to the constraints

$$x_1 + 5x_2 \leq 10$$
$$x_1 + 3x_2 \geq 6$$
$$2x_1 + 2x_2 \leq 8$$

$x_2 \geq 0$ **and** x_1 **unrestriced.**

The dual must have exactly two constraints and three variables.

Sol. Formulate the dual of the following L.P.P.

Maximize $Z = 2x_1 + x_2$

Subject to the constraints

$$x_1 + 5x_2 \leq 10$$
$$x_1 + 3x_2 \geq 6$$
$$\Rightarrow \quad -x_1 - 3x_2 \leq -6$$
$$2x_1 + 2x_2 \leq 8$$

$x_2 \geq 0$ and xy unrestricted.

Minimize $y = 10y_1 - 6y_2 + 8y_3$

Subject to the constraints

$$y_1 - y_2 + 2y_3 \geq 2$$
$$5y_1 - 3y_2 + 2y_3 \geq 1$$
$$y_2 \geq 0, y_3 \geq 0$$

and y_1 is unrestricted.

***(b)* Solve graphically the following LPP:**

Maximize $z = 3x_1 + 2x_2$

Subject to the constraints

$$-2x_1 + x_2 \leq 1$$
$$x_1 \leq 2$$
$$x_1 + x_2 \leq 3$$
$$x_1, + x_2 \geq 0.$$

Ans. Maximize $z = 3x_1 + 2x_2$

Subject to the constraints.

$$2x_1 + x_2 \leq 1$$
$$x_1 \leq 2$$
$$x_1 + x_2 \leq 3$$
$$x_1, x_2 \geq 0$$

First, write the inequality of the constraints into an equation and plot the line in the graph.

$2x_1 + x_2 = 1$ passes through (0, 1) (½, 0)

$x_1 = 2$ passes through (2, 0)

$x_1 + x_2 = 3$ passes through (0, 3) (3, 0)

We marks the region below the lines lying in the first quadrant as the inequality of the constraints are $\leq$.

The feasible region is ABCDE

Corner points	Values of $z = 3x_1 + 2x_2$
A (½, 0)	Max $Z_A = 3/2 = 1.5$
B (0, 1)	$Z_B = 2$
C (0, 3)	$Z_C = 6$
D (2, 1)	$Z_D = 8$
E (3, 0)	$Z_E = 6$

Sample Preview
of
The Chapter

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LINEAR PROGRAMMING

Basic Mathematics and Optimization

Basic Algebra

1

INTRODUCTION

Algebra is the branch of mathematics that uses letters in place of some unknown numbers. During your early schooling, you must have learnt formulas in algebraic equation like area of a rectangle: $A = w \times h$. The letters denote some unknown numbers, once we knew the value of width and height; we could substitute them into the formula. Thus, algebra is the study of the properties of operations on numbers. It is a problem solving tool. The linear equation is an algebraic equation in which each term is either a constant or the product of a constant and a single variable. Linear equations can have one or more variables. Various elementary methods are used to solve these linear equations which involve two or three variables. But these methods are not helpful where a large number of equations involve more than three variables. Thus, few other methods are involved for such equations; such as Matrices and Determinants. By the end of this unit, you will be able to define a matrix and a determinant, addition and multiplication of two matrix, obtain the determinant of a matrix, and compute the inverse of a matrix, Matrices and Determinants.

CHAPTER AT A GLANCE

MATRICES AND DETERMINANTS

Matrix: A matrix is an ordered rectangular array of numbers. The numbers or entries are called the elements of the matrix. The rectangular arrays of entries are enclosed in an ordinary bracket or in a square bracket. They can be used to represent systems of linear equations. Consider the following arrangement:

$$\begin{bmatrix} 3 & 4 & 2 \\ 6 & 8 & 9 \end{bmatrix}$$

In this arrangement, there are two rows and three columns. The number 8 lies in the 2nd row and 2nd column.

$$A = \begin{bmatrix} 4 & 7 \\ 9 & 5 \\ 3 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 10 & 4 \\ 14 & 17 \end{bmatrix}$$

In the above arrangement, matrix Ahas 3 rows and 2 columns and the order is 3 × 2. Matrix B has 2 rows and 2 columns and the order is 2 × 2. Capital letters are used to denote the matrix. The system of linear equations can also converted into matrix:

$$3x - 5y + z = 6$$
$$4x + y - z = 8$$
$$x + y - z = 1$$

$$A = \begin{bmatrix} 3 & -5 & 1 \\ 4 & 1 & -1 \\ 1 & 1 & -1 \end{bmatrix}$$

Definition of a Matrix

- Rectangular array of numbers.
- A matrix with m rows and n columns is called a matrix of order $m \times n$.
- First subscript is row and second subscript is column.
- A row matrix is a matrix having only one row.
- A column matrix is a matrix having one column.
- If the number of rows and columns are same in a matrix, it is known as a square matrix.

$$A^{m \times n} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \ldots\ldots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \ldots\ldots. & a_{2n} \\ a_{31} & a_{32} & a_{33} & \ldots\ldots. & a_{3n} \\ . & . & . & \ldots\ldots. & . \\ . & . & . & \ldots\ldots. & . \\ a_{m1} & a_{m2} & a_{m3} & \ldots\ldots. & a_{mn} \end{bmatrix}$$

Equal Matrix

Two matrix can be equal if and only if these matrix have the same dimensions and equal corresponding elements. Let's consider following two matrix; named A and B:

$$A = \begin{bmatrix} -5 & 1 & 8 \\ 1 & -6 & -2 \\ 9 & 4 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} -5 & 1 & 8 \\ 1 & -6 & -2 \\ 9 & 4 & 2 \end{bmatrix}$$

Are these matrix equal? Yes, because

- Both the matrix have the same dimensions i.e. 3 rows and 3 columns (3 × 3)
- All the corresponding elements are same in both matrix.

Example: Which of the following matrix are equal?

$$A = \begin{bmatrix} 3 & 1 & 18 \\ 1 & -2 & -12 \\ 5 & 8 & -6 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & 1 & 18 \\ 1 & -4 & -12 \\ 5 & 8 & 6 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & 1 & 18 \\ 1 & -2 & -12 \\ 5 & 8 & -6 \end{bmatrix}$$

Matrix A and C are equal. Since they have same dimensions and equal corresponding elements.

Square Matrix

A square matrix is a matrix which has the same number of rows and columns. Consider the following matrix:

$$A = \begin{bmatrix} -2 & -8 & 4 \\ 4 & 9 & -13 \\ 3 & 1 & 6 \end{bmatrix}$$

It is a square matrix, as it is having 3 rows and 3 columns (3 × 3).

Let us consider following matrix:

$$A = \begin{bmatrix} 0 \\ 4 \\ 17 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 4 & 1 & 1 \end{bmatrix}$$

The A matrix is in order of 3 × 1 (3 rows and 1 column) and B matrix is in order of 1 × 4 (1 row and 4 columns). **When a matrix is having only one column is called a column matrix**, thus A is a column matrix and **when a matrix is having only one row is called a row matrix**, thus B is a row matrix. Consider the following matrix:

$$A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

It is a square matrix having all elements as zero except the diagonal elements. Such types of matrix are known as **Diagonal Matrices.** The diagonal elements are: 4, 9, and 5.

Identity Matrix: An Identity matrix is a diagonal matrix with all diagonal element = 1.

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Zero Matrix: When all the elements of a matrix are 0, then that matrix is called a Zero matrix or Null matrix. For example.

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= B \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$$

Submatrix

A sub matrix is formed by selecting certain rows and columns from a bigger matrix or if we delete few rows and columns from a matrix, the resulting matrix is called a sub matrix.

$$A = \begin{bmatrix} 2 & 8 & 18 & 4 \\ 9 & -2 & 13 & 3 \\ 5 & 6 & 9 & 1 \\ 1 & 3 & 7 & 9 \end{bmatrix}$$

If we delete third row and second column, we obtain the following matrix:

$$B = \begin{bmatrix} 2 & 18 & 4 \\ 9 & 13 & 3 \\ 1 & 7 & 9 \end{bmatrix}$$

Scalar Multiplication

In scalar multiplication, each element of the matrix is multiplied by the number or scalar. For example, if

$$A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \\ 6 & 7 \end{bmatrix}, \text{ determine } 3A.$$

Solution:

$$A = \begin{bmatrix} 2\times3 & 4\times3 \\ 1\times3 & 3\times3 \\ 6\times3 & 7\times3 \end{bmatrix} = \begin{bmatrix} 6 & 12 \\ 3 & 9 \\ 18 & 21 \end{bmatrix}$$

Addition of Matrix

If A and B are two matrix of the same size. Then their sum C = A + B is defined by the following rule:

$$C_{ij} = A_{ij} + B_{ij}$$

Example:

$$A = \begin{bmatrix} 3 & 5 & 9 \\ 5 & 1 & 3 \\ -2 & 4 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 1 & 0 \\ 3 & 6 & -4 \\ 5 & 3 & 1 \end{bmatrix}$$

Find A + B.

Solution:

$$A = \begin{bmatrix} 3+2 & 5+1 & 9+0 \\ 5+3 & 1+6 & 3+(-4) \\ -2+5 & 4+3 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 6 & 9 \\ 8 & 7 & -1 \\ 3 & 7 & 1 \end{bmatrix}$$

Difference of Matrix

If A and B are two matrix of the same size. Then their difference C = A – B is defined by the following rule:

$$C_{ij} = A_{ij} - B_{ij}$$

Example:

$$A = \begin{bmatrix} 4 & 3 & 1 \\ 6 & -2 & 9 \end{bmatrix}$$

$$B = \begin{bmatrix} -3 & 2 & 0 \\ 1 & 3 & 5 \end{bmatrix}$$

Find A – B.

Solution:

$$A = \begin{bmatrix} 4-(-3) & 3-2 & 1-0 \\ 6-1 & -2-3 & 9-5 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 1 & 1 \\ 5 & -5 & 4 \end{bmatrix}$$

Product of Matrix

When the number of columns of the first matrix is the same as the number of rows in the second matrix then product of matrix can be performed.

Matrix multiplication for 2 × 2 matrix:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$$

Matrix multiplication for 3 × 3 matrix:

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} j & k & l \\ m & n & o \\ p & q & r \end{bmatrix}$$

$$= \begin{bmatrix} aj+bm+cp & ak+bn+cq & al+bo+cr \\ dj+em+fp & dk+en+fq & dl+eo+fr \\ gj+hm+ip & gk+hn+iq & gl+ho+ir \end{bmatrix}$$

Example:

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 5 & 4 \\ 6 & 8 & 7 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 5 & 8 \\ 4 & 3 & 2 \end{bmatrix}$$

Find the product of two matrix i.e. A × B.

Sol.

$$\begin{bmatrix} 2 & 3 & 1 \\ 3 & 5 & 4 \\ 6 & 8 & 7 \end{bmatrix} \times \begin{bmatrix} 3 & 2 & 6 \\ 1 & 5 & 8 \\ 4 & 3 & 2 \end{bmatrix}$$