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LINEAR ALGEBRA

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QUESTION PAPER

(June – 2016)

(Solved)

LINEAR ALGEBRA

Time: 2 hours] *[Maximum Marks: 50*
(Weightage 70%

Note: *Question no. 7 is **compulsory**. Attempt any **four** questions from Questions no. 1 to 6. Use of calculators is **not** allowed.*

Q. 1. *(a)* Let $\alpha_1 = (1, 0, -1), \alpha_2 = (1, 2, 1)$ and $\alpha_3 = (0, -3, 2)$ be vectors in R^3. Show that $\{\alpha_1, \alpha_2, \alpha_3\}$ is a basis for R^3. Express (1, 0, 0) and (1, 1, 0) as linear combinations of α_1, α_2 and α_3.

***(b)* Let $T : R^3 \to R^3$ be defined by**

$T(x_1, x_2, x_3) = (3x_1 + x_3, -2x_1 + x_2, -x_1 + 2x_2 + 4x_3)$

Sol. To check if $\{(1, 0, -1), (1, 2, 1) (0, -3, 2)\}$ is linearly independent over R^3.

Let $a(1, 0, -1) + b(2, 3, 1) + c(3, 1, 2) = (0, 0, 0)$

$(a, 0, -a) + (2b, 3b, b) + (3c, c, 2c) = (0,0,0)$

$$a + 2b + 3c = 0 \quad ...(i)$$
$$3b + c = 0 \quad ...(ii)$$
$$-a + b + 2c = 0 \quad ...(iii)$$

Equation *(i)* and *(iii)* adding, we get

$$3b + 5c = 0 \quad (iv)$$

From equation (ii) and (iv), we get

$$c = 0$$

Put the value of c in equation *(ii)* we get

$$b = 0$$

Now putting the value of b & c in equation *(i)* we get

$$a = 0$$

So that the given set $\{\alpha_1, \alpha_2, \alpha_3\}$ is a basis for R^3.

Suppose $S = \{(1, 0, 0) (1, 1, 0)\}$

$$[S] = \{\alpha(1, 0, 0) + \beta(1, 1, 0)\} | \alpha, \beta, \in R\}$$
$$= \{\alpha + \beta\}.(\beta) | \alpha, \beta \in R\}$$

Then $\{(1, 0, 0)$ and $(0,1, 0)\} \notin [S]$

So that (1, 0,0) and (1, 1, 0) are two distinct bases of R^3. {(1, 0, 0), (0,1,0) (1, 0, 0)} and {(1, 0, 0) (0, 1, 0) (0, 1, 0}

***(i)* Write the matrix of T with respect to the standard basis of R^3.**

Sol. Given $T : R^3 \to R^3$.

$$T(x_1 x_2 x_3) = (3x_1 + x_3, -2x_1 + xz_j - x_1 + zx_2 + 4x_3)$$

$$B_1 = (e_1, e_2 e_3)$$

$B_2 = \{f_1, f_2, f_3\}$ are standard basis.

$$T(e_1) = T(1, 0, 0)$$
$$= (3, -2, -1)$$
$$= 3f_1 - zf_2 - f_3$$
$$T(e_1) = T(0, 1, 0)$$
$$= (0, 1, 2)$$
$$= f_2 + 2f_3$$
$$T(e_3) = T(0, 0, 1)$$
$$= (1, 0, 4)$$
$$= f_1 + 4f_3$$

***(ii)* Show that T^{-1} exists. Give the expression for $T^{-1}(x_1, x_2, x_3)$ for T above.**

Sol. $T^{-1}(x_1, x_2, x_3) = (3x_1 + x_3, -2x_1 + x_2, -x_1 + 2x_2 + 4x_3)$

$$T^{-1}(e_1) = \frac{1}{T}(1, 0, 0)$$

$$T^{-1}(e_1) = \frac{1}{(3f_1 - 2f_2 - f_3)}$$

$$T^{-1}(e_2) = \frac{1}{(f_1 + 2f_3)}$$

$$T^{-1}(e_2) = \frac{1}{(f_1 + 4f_3)}$$

Q. 2. *(a)* Let $F : R^2 \to R^1$ be defined by $f(x_1, x_2) = 3x_1 + 4x_2$ and $T : R^2 \to R^2$ be defined by $T(x_1, x_2) = (x_1 - x_2, x_1 + x_2)$.

Suppose g = f oT. What is g (2, 3) ?

Sol. $f(x_1, x_2) = 3x_1 + 4x_2$

$$T(x_1, x_2) = (x_1 - x_2, x_1 + x_2)$$
$$g(2,3) = f_0 T$$
$$g(2,3) = f[T(x_1, x_2)]$$
$$g(2,3) = f[x_1 - x_2, x_1 + x_2]$$
$$g(2,3) = [3(x_1 - x_2) + 4(x_1 + x_2)]$$
$$g(2,3) = 3x_1 - 3x_2 + 4x_1 + 4x_2$$
$$g(2,3) = 7x_1 + x_2$$
$$g(2,3) = 7 \times 2 + 3$$
$$g(2,3) = 14 + 3$$
$$g(2,3) = 17$$

***(b)* Let** $A = \begin{bmatrix} a & 2 & 1 \\ 0 & b & 1 \\ 1 & 1 & 1 \end{bmatrix}$

***(i)* Find one value each of *a* and *b* such that rank of A is 3. Justify your answer.**

Sol. If rank of A is 3 then

$$A = \begin{bmatrix} a & 2 & 1 \\ 0 & b & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

taken value of a & b . so that

$$\begin{bmatrix} 2 & 1 \\ b & 1 \end{bmatrix} = 0$$
$$2 - b = 0$$
$$\boxed{b = 2}$$

and $\begin{bmatrix} a & 1 \\ 0 & 1 \end{bmatrix} \Rightarrow \boxed{a = 0}$

So that value of *a* and *b* is (0, 2).

***(ii)* Find one value each for *a* and *b* such that rank of A is 2. Justify your answer.**

(ii) If rank of A is 2, so reduce the Given matrix is form of rank of A is 2.

$$A = \begin{bmatrix} a & 2 & 1 \\ 0 & b & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$R_2 \to R_3 - R_2$ and $R_3 \to R_3 - R_1$

$$A = \begin{bmatrix} a & 2 & 1 \\ -1 & b-1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} a & 2 & 1 \\ -1 & b-1 & 0 \\ a-1 & 1 & 0 \end{bmatrix}$$

So matrix is in form of Rank 2.

$$\begin{bmatrix} 2 & 1 \\ b-1 & 0 \end{bmatrix} = 0 \quad \text{and} \quad \begin{bmatrix} 9 & 1 \\ -1 & 0 \end{bmatrix}$$
$$0 = b - 1 \text{ and } 0$$
$$b = 1$$

So that value of $b = 1$ in form of matrix A, if rank is 2.

***(c)* Find the minimal polynomial of the matrix**

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Sol. Given matrix $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$$f_A(t) = \begin{bmatrix} t & 1 & 0 \\ 0 & t & 0 \\ 0 & 0 & t \end{bmatrix}$$
$$= t\begin{bmatrix} t & 0 \\ 0 & t \end{bmatrix} - 1\begin{bmatrix} 0 & 0 \\ 0 & t \end{bmatrix} + 0$$
$$= t[t^2 - 0] - 1[0] + 0$$
$$= t^3$$

So Minimal polynomial can be t^3.

Q. 3. *(a)* Find the eigen values and eigen vectors of the matrix

$$A = \begin{pmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{pmatrix}$$

Sol. Given matrix $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$

Then

$$f_A(t) = \begin{bmatrix} t-5 & -6 & -6 \\ -1 & t-4 & 2 \\ 3 & -6 & t+4 \end{bmatrix}$$
$$= (t-5)[(t-4)(t+4) + 12]$$

Sample Preview
of
The Chapter

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LINEAR ALGEBRA

VECTOR SPACES

Sets, Functions and Fields

1

Sets

Definition: A well-defined collection of objects (even ideas) is a set.

We mean 'Mathematically measurble' or distinguishable by word 'well-defined.'

There are two ways of describing a set, *viz.,*

(1) **Roster Method:** Just listing elements in Parenthesis.

(2) **Set Builder Method:** Rule form.

Subsets: A set A is said to be subset of another set B if every element of A is also an element of set B. In this case, we write $A \subseteq B$ to say that A is subset of B. Here B is called superset of A, written as $B \supseteq A$.

Union of Sets: Union of two (or more) sets is the set which contains all elements of A as well as B. It is denoted by $A \cup B$.

Intersection of Sets: Let A, B be two sets then intersection of A and B written as $A \cap B$ is the set containing common elements of A and B.

Above definitions of union and intersection can be easily extended to three or more sets.

Empty Set: A set having no element written as ϕ or {} is called empty set.

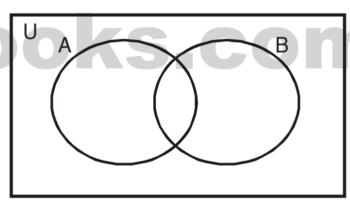
Note that {0} or {ϕ} are not empty sets.

Universal Set: A set U is called universal set if it is superset of all sets under consideration.

Complement of a Set: Let A be a set and U, the universal set then set U-A written as A^C or A', is complement of set A.

Note that A^C contains all elements of U which do not belong to A.

Venn Diagrams: A universal set will be taken as a rectangle and all other sets usually as circles, e.g. following diagram.



Cartesian Product of Sets

Definition: A Cartesian product $A \times B$ of the sets A and B is the set of all possible ordered points (a, b) where, $a \in A$ and $b \in B$.

Let A, B be two sets then their cartesian product written as $A \times B$ is given by $A \times B = \{(x, y) ; x \in A, y \in B\}$.

Relations

A relation R on a set S is a relationship between elements of S, where $R \subseteq S \times S$.

Let $R: A \rightarrow B$ is a relation,

(1) If $a\ Ra$ i.e., every element $a \in S$ is image of itself, we say R is reflexive.

(2) If $a\text{R}b \Rightarrow b\text{R}a$ then R is called symmetric $\forall\, a, b \in \text{S}$.

(3) If $a\text{R}b$ and $b\text{R}c \Rightarrow a\text{R}c$ then we say that R is transitive,

$$\forall\, a, b, c \in \text{S}.$$

Note that for R to be defined, S has to be non-empty.

If R has all three properties, viz., reflexive, symmetric and transitive, then we say that R is an equivalence relation.

Functions

Functions are special types of relations. Let us denote it by f i.e., $f: \text{A} \to \text{B}$ where A is called domain and B is co-domain of f.

Range of f is given by:

$$\{f(a) : a \in \text{A}\}$$

Note that in function $f: \text{A} \to \text{B}$,

(1) For each and every element of A, there is some element of B associated.

(2) For each element of A, we associate only one element of B.

(3) Two or more elements of A could be associated with one element of B.

Definition: A function $f: \text{A} \to \text{B}$ is a relation such that it associates with every element of A, exactly one element of B.

A function $f: \text{A} \to \text{B}$ is said to be one-one (or 1– 1) or injective if each element of A are associated with different elements of B.

A function $f: \text{A} \to \text{B}$ is said to be onto (or surjective) if the range of f is B.

If function f has both properties of 1– 1 and onto, we call it bijective.

Composition of Functions: If $f: \text{A} \to \text{B}$ and $g: \text{C} \to \text{D}$ are functions and if the range of f is a subset of C, there is a natural way of combining g and f, called $go\,f$ such that:

$$(go\,f)\,(x) = g[f(x)]$$

Similarly,

$(fog)\,(x) = f\,[g(x)]$, if fog is defined.

Binary Operation: Let S be a non-empty set then a binary operation is a function from S ×S to S.

Definition: A binary operation on a set S is said to be:

(1) Closed on subset T of S if

$$t_1, t_2 \in \text{T}\ \forall\, t_1, t_2 \in \text{T}$$

(2) Commutative if $ab = ba\ \forall a, b \in \text{S}$.

(3) Associative if $(ab)\,c = a(bc)$

$$\forall a, b, c \in \text{S}.$$

We shall have further more the distributive property for two binary operations, say + and · on R, such that

$$a.\,(b+c) = a.b + a.c\ \ \forall a, b, c \in \text{R}.$$

Fields

Let two binary operations + and · on a non-empty set F be defined. If F has following 9 properties in relation to + and·, then we call it a field.

A-1: $(a + b) + c = a + (b + c),\ \forall a, b, c \in \text{F}$

(Associativity)

A-2: $\exists$ an identity element, 0 such that

$a + 0 = 0 + a = a \qquad \forall a \in \text{F}$

(Existence of zero)

A-3: For every element of $a \in \text{F}$, $\exists\, b$, written as $b = -a$ s.t.

$$a + b = b + a = 0 \text{ in F}.$$

(Additive Inverses)

A-4: $a + b = b + a \qquad \forall a, b \in \text{F}$

(Commutativity)

M-1: $(a.b)\,c = a.\,(b.c)\ \ \forall a, b, c \in \text{F}$.

M-2: $\exists\, e \in \text{F}$ s.t.

$$a.\,e = e.\,a,\ \forall a \in \text{F}.$$

e is called multiplicative identity.

M-3: For all $a \in \text{F} - \{0\}$, $\exists\ b\left(= \frac{1}{a}\right)$ in F s.t

$$a.b = e = b.a$$

b is also written as a^{-1}

M-4: $a.b = b.\,a$ (Commutativity)

D-1: Distribution of over +

$a.(b + c) = a.b + a.c \quad \forall a, b, c \in \text{F}.$

EXERCISE QUESTIONS

Example 1. Write the following sets by the roster method.

A = $\{x \mid x$ is an integer and $10 < x < 15\}$
B = $\{x \mid x$ is an even integer and $10 < x < 15\}$
C = $\{x \mid x$ is a positive divisor of $20\}$
D = $\{p/q \mid p, q$ integers and $1 \le P < q \le 3\}$

Sol. A = {x/x is an integer and $10 < x < 15$ }

$\therefore$ A = {11, 12, 13, 14}

B = {x/x is an even integer and $10 < x < 15$}

$\therefore$ B = {12, 14 }

C = {x/x is a positive divisor of 20}

$\therefore$ C = {1,2,4,5,10, 20}

$$D = \left\{ \frac{p}{q} / \, p, q \text{ are integers and} 1 \leq p < q \leq 3 \right\}$$

$$= \left\{ \frac{1}{2}, \frac{1}{3}, \frac{2}{3} \right\}$$

Example 2. Write the following sets by the set builder method.

P = {7, 8, 9}; Q = {1, 2, 3, 5, 7, 11};

R = {3, 6, 9,.......}.

Sol. P = {7,8,9}

$\therefore$ P = {x: $x \in$N, $6 < x < 10$},

Q = {1,2,3,5,7,11}

$\therefore$ Q = { x : x is a prime number ≤ 11}

R = {3,6,9,........}

$\therefore$ R = {x : $x = 3y$, $y \in$N}.

Example 3. Which of the following statements are true?

(a) **N ⊆ Z** ***(b)*** **Z ⊆ N**

(c) **{0} ⊆ {1, 2, 3}** ***(d)*** **{2, 4, 6} ⊄ {2, 4, 8}**

Sol. *(a)* N = { 1,2,3,4......}

Z = {....., –3, –2, –1, 0,1,2,3,.....}

$\therefore$ N ⊆ Z is true.

(b) Obviously Z ⊆ N is false.

(c) {0} ⊆ {1,2,3} is false.

(d) {2,4,6} ⊈ {2,4,8} is a true statement.

Example 4. Show that, if A ⊆ C and B ⊆ C, then A ∪ B ⊆ C.

Sol. Given A ⊆ C and B ⊆ C

Then prove that

A ∪ B ⊆ C

Let $x \in$ A $\Rightarrow x \in$ C as A ⊆ C

Also $x \in$ B $\Rightarrow x \in$ C as B ⊆ C

$\therefore$ $x \in$ A or $x \in$ B

$\Rightarrow$ $x \in$ A ∪ B

$\Rightarrow$ $x \in$ C

$\therefore$ A ∪ B ⊆ C. Hence proved.

Example 5. For every set A, show that ϕ ∪ A = A and ϕ ∩ A = ϕ.

Sol. For any set A, to prove that

ϕ ∪ A = A and ϕ ∩ A = ϕ

We consider ϕ ∪ A

= {$x \in$ ϕ or $x \in$ A}

= { x : $x \in$ A}, as ϕ = { }

$\therefore$ ϕ ∪ A = A

Next ϕ ∩ A = { x : $x \in$ ϕ and $x \in$ A}

= {x : $x \in$ ϕ}

$\therefore$ ϕ ∩ A = ϕ Hence proved.

Example 6. State whether are following are true or false.

(a) **If A Í B and B Í C, then A Í C.**

(b) **If A ⊈ B and B ⊈ A, then A and B are disjoint.**

(c) **A ⊈ (A ∪ B)**

(d) **B ⊆ (A ∪ B)**

(e) **If A ∪ B = ϕ then A = B = ϕ.**

Sol. *(a)* Given A ⊆ B and B ⊆ C then to prove that A ⊆ C or otherwise

Let $x \in$ A

$\Rightarrow$ $x \in$ B as A ⊆ B

$\Rightarrow$ $x \in$ C as B ⊆ C.

$\therefore$ A ⊆ C Hence, it is True

(b) Given A ⊈ B and B ⊈ A

then to prove that or otherwise,

A, B are disjoint.

If $x \in$ A $\Rightarrow$ $x \notin$ B as A ⊈ B

If $y \in$ B $\Rightarrow$ $y \notin$ A as B ⊈ A

$\therefore$ $x \in$ A ∪ B $\Rightarrow$ $x \in$ A or $x \in$ B or $x \in$ A ∩ B

$\Rightarrow$ $x \in$ A ∩ B

So, A, B are not disjoint.

$\therefore$ Given statement is false.

(c) To find true or false

Given statement.

A ⊈ (A ∩ B)

Let $x \in$ A

$\Rightarrow \quad x \in A$ or $x \in B$

$\Rightarrow \quad x \in A \cup B$

$\therefore \quad A \subseteq (A \cup B)$ is true.

And hence $A \not\subseteq A \cup B$ is false.

(d) Given statement $B \subseteq (A \cup B)$

Let $\quad x \in B$

$\Rightarrow \quad x \in A$ or $x \in B$

$\Rightarrow \quad x \in A \cup B$

$\therefore \quad B \subseteq A \cup B$ is true

(e) Given $A \cup B = \phi$

Let $\quad x \in A \cup B$

$\Rightarrow \quad x \in A$ or $x \in B$

$\Rightarrow \quad x \in \phi$ or $x \in \phi$

$\therefore$ i.e. $\quad A = \phi, B = \phi$

So $A \cup B = \phi \Rightarrow A = B = \phi$ is true.

Example 7. Suppose A = {*a*, *b*, *c*} B = {*a*, *b*, *p*, *q*} and C = {*a*, *p*, *r*, *s*}. Find the following sets:

(a) **$A \cup B$,** ***(b)*** **$B \cap C$,**

(c) **$(A \cup B) \cap C$,** ***(d)*** **$(A \cap C) \cup (B \cap C)$.**

Sol. Given

$$A = \{a, b, c\}$$
$$B = \{a, b, p, q\}$$
$$C = \{a, p, r, s\}$$

(a) $\quad A \cup B = \{a, b, c, p, q\}$

(b) $\quad B \cup C = \{a, p\}$

(c) $\quad (A \cup B) \cap C = \{a, b, c, p, q\} \cap \{a, p, r, s\}$

$\therefore \quad (A \cap B) \cap C = \{a, p\}$

(d) $\quad A \cap C = \{a\}$

$\quad B \cap C = \{a, b\}$

$\therefore \quad (A \cap C) \cup (B \cap C) = \{a, p\}$.

Example 8. Why are the following statements true?

(a) **A and A^c are disjoint, i.e., $A \cap A^c = \phi$.**

(b) **$A \cup A^c = X$, where X is the universal set.**

(c) **$(A^c)^c = A$**

Sol. *(a)* Let $x \in A$

$\Rightarrow \quad x \notin A^c$

$\therefore \quad A \cap A^c = \phi$ True.

(b) We know $A^c = X - A$

Also $\quad A \cup A^c = X$

$\Rightarrow \quad x \in X$

$\Rightarrow \quad x \in A$ or $x \in A^C$

$\Rightarrow \quad x \rightarrow A \cup A^c$

$\therefore \quad A \cup A^c = X$ True.

(c) To check $(A^c)^c = A$

$\Rightarrow \quad x \in (A^c)^c$

$\Rightarrow \quad x \notin A^c$

$\Rightarrow \quad x \in A$

$\therefore \quad (A^c)^c \subseteq A \quad ...(1)$

Next let $\quad x \in A$

$\Rightarrow \quad x \notin A^c$

$\Rightarrow \quad x \in (A^c)^c$

$\therefore \quad A \subseteq (A^c)^c \quad ...(2)$

By (1) and (2)

$(A^c)^c = A$. Proved.

Example 9. Try and prove $(A \cap B)^C = A^C \cup B^C$

Sol. To prove that

$$(A \cap B)^c = A^c \cup B^c$$

Let $\quad x \in (A \cap B)^c$

$\Leftrightarrow \quad x \notin A \cap B$

$\Leftrightarrow \quad x \notin A$ or $x \notin B$

$\Leftrightarrow \quad x \in A^c$ or $x \in B^c$

$\Leftrightarrow \quad x \in A^c \cup B^c$

$\therefore \quad (A \cap B)^c = A^c \cup B^c$ **Hence proved.**

Example 10. Use Venn diagrams to demonstrate the truth of the following results. Here A, B, C are subsets of X.

(a) **(A È B)Ç C = (A Ç C)È (B Ç C)**

(b) **(A Ç B)È C = (A È C)Ç (B È C)**