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MATHEMATICAL METHODS

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Published by:

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MATHEMATICAL METHODS

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Sample Preview of the Solved Sample Question Papers

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QUESTION PAPER

(June – 2017)

(Solved)

MATHEMATICAL METHODS

Time: 2 hours] *[Maximum Marks: 50*
(Weightage 70%)

Note: *Question no. 7 is* ***compulsory****. Attempt any* ***four*** *questions from Q. No. 1 to Q. No. 6. Use of calculators is* ***not*** *allowed.*

Q. 1. ***(a)*** **Evaluate:**

$$\lim_{x \to 1} \frac{x^2 - 1}{\sqrt{3x+1} - \sqrt{5x-1}}$$

Sol. $\lim_{x \to 1} \frac{x^2 - 1}{\sqrt{3x+1} - \sqrt{5x-1}}$

Put $x = 1$

$$= \frac{(1)^2 - 1}{\sqrt{3 \times 1 + 1} - \sqrt{5 \times 1 - 1}}$$

$$= \frac{1 - 1}{\sqrt{4} - \sqrt{4}}$$

$$= 0$$

(b) **Find the centre and radius of the sphere $4x^2 + 4y^2 + 4z^2 - 48x - 36y + 16z + 205 = 0$.**

Sol. Given Equation

$$4x^2 + 4y^2 + 4z^2 - 48x - 36y + 16z + 205 = 0$$

compairing with sphere equation

$$x^2 + y^2 + z^2 - 2ux - 2vy - 2wz + d = 0$$

then $d = 205$

$w = -8$

$v = 18$

$u = 24$

centre point is $(u, v, w) = (24, 18, -8)$ and

radius $= \sqrt{u^2 + v^2 + w^2 - d^2}$

$= \sqrt{576 + 324 + 64 - 42025}$

$= \sqrt{41061}$

$= 202.63$

(c) **Find the points of maxima or minima of the function $f(\theta) = a \cos^2 \theta + b \sin^2 \theta$ $(a > b)$.**

Sol. $f(\theta) = a \cos^1 \theta + b \sin^2 \theta \ (a > b)$

$\therefore f'(\theta) = 2a \cos \theta (-\sin \theta) + 2b \sin \theta \cos \theta$

$= 2a \sin \theta \cos \theta + 2b \sin \theta \cos \theta$

$f'(\theta) = 2 \sin \theta \cos \theta \ (b - a)$

$= \sin 2\theta \ (b - a)$

Putting

$f'(\theta) = \sin 2\theta \ (b - a) = 0$ For Maximum

$\Rightarrow$ $\sin 2\theta = 0 = \sin \theta$ or $\sin 2x\pi$

$\therefore \quad 2\theta = 0, \pm 2x\pi$

$\theta = 0, \pm x\pi, x \in_1$

After differentiating

$f'(\theta) = (b - a) \sin 2\theta$

$f'(\theta) = 2 (b - a) \cos 2\theta$

$f''(\pm x\pi) = -2 (a - b) \cos 2x\pi < 0$

$\therefore$ Minimum exists at $0, \pm x\pi$

Q. 2. ***(a)*** **Evaluate:**

$$\mathbf{I = \int_0^{\pi/2} \frac{\sin x}{1 + \cos^2 x} dx}$$

Sol.

$$I = \int_0^{\pi/2} \frac{\sin x}{1 + \cos^2 x} dx$$

$$I = \int_0^{\pi/2} \frac{\sin x}{1 + u^2} \cdot \frac{du}{-\sin x}$$

where $u = \cos x$

$du = -\sin x \, dx$

$dx = \frac{du}{-\sin x}$

$$I = -\int_0^{\pi/2} \frac{1}{1 + u^2} du.$$

$$I = -\left[\tan^{-1}\mu\right]_0^{\pi/2}$$

$$I = \left[-\tan^{-1}\cos x\right]_0^{\pi/2}$$

$$I = -\tan^{-1}\cos\pi/2 - \tan^{-1}\cos 0$$

$$I = -\tan^{-1}0 - \tan^{-1}.1$$

$$I = -\tan^{-1}(1)$$

(b) **In a class of 10 boys and 20 girls, half the boys and half the girls have black eyes. What is the probability that a student chosen at random is a boy or has black eyes?**

Sol. From half the boys (10/2 = 5), a student chosen at random is a boy or has black eyes $5c_1$ ways.

$$\therefore \quad \text{probability} = {}^5c_1$$

$$= \frac{\lfloor 5}{\lfloor 4 \lfloor 1}$$

$$= 5 \text{ ways.}$$

(c) **If $f: R \to R$ and $g: R \to R$ are defined by $f(x) = \frac{x+1}{2}$ and $g(x) = 2x^2 - 1$, find $fog(x)$. In the function fog injective, surjective or monotone?**

Sol. $f(x) = \frac{x+1}{2}$,

$$g(x) = 2x^2 - 1$$

$$fog(x) = fg(x)$$

$$fog(x) = f(2x^2 - 1)$$

$$fog(x) = \frac{2x^2 - 1 + 1}{2}$$

$$fog(x) = x^2$$

suppose $x = 1$ then

$$f(x) = \frac{1+1}{2} = 1, \; g(x) = 2 - 1 = 1$$

So that $f(x)$ is injective and surjective.

Q. 3. ***(a)*** **Calculate the medium height of the students from the following data:**

Height (in inches)	No. of Students
60–63	25
63–66	108
66–69	154
69–72	115
72–75	18

Sol.

Height C.I.	frequency f_i	class values x_i	c.f.	$x_i.f_i$
60-63	25	61.5	25	1537.5
63-66	108	64.5	133	6966
66-69	154	67.5	287	10395
69-72	115	70.5	402	8107.5
72-75	18	73.5	420	1323
	$n = 420$			$\sum f_i x_i = 28329$

$$\frac{n}{2} = \frac{420}{2} = 210$$

$\therefore$ Median lies in 66 – 69

$l = 66$

$c = 133$

$t = 154$

$i = 4$

$$\text{Median} = l + \frac{\frac{n}{2} - c}{f} \times i$$

$$= 66 + \frac{210 - 133}{154} \times 4$$

$$= 66 + 2$$

$$\boxed{\therefore \text{Median} = 68}$$

(b) **Solve:**

$$\frac{dy}{dx} + y \tan x = \sec x$$

Sol. $$\frac{dy}{dx} + y \tan x = \sec x$$

Comparing with $\frac{dy}{dx} + P_y = \theta$

we get $P = \tan x, \; \theta = \sec x$

$\therefore$ $I.F. = e^{\int P dx}$

$$= e^{\int \tan x \, dx}$$

$$= e^{\log \sec x}$$

$$= \sec x$$

$y (\sec x)^{-1} [\sec x \sec x \, dx + c]$

$$y = \frac{1}{\sec x} \int \sec x \, dx + c$$

$$y = \frac{1}{\sec x} \cdot \frac{1}{\cos x} + c$$

$$y = \frac{1}{\cos x \sec x}$$

Sample Preview of The Chapter

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MATHEMATICAL METHODS

ALGEBRA AND GEOMETRY

SETS AND FUNCTIONS

1

SETS

A set is taken as "a well-defined collection of objects." What is well-defined? "Well-defined" means distinguishable and mathematically (at least theoretically) measurable.

There are basically two ways of expressing a set, viz.

1. **Roaster Form:** Just listing the elements or members in parenthesis, { }, e.g.
   A = {*a*, *b*, *c*}, A is a set.
2. **Rule Form or Set Builder Form:** When some property (or rule) of every element describes the set, e.g., A = {*a*, *b*, *c*} can be taken as
   A = {*x* : *x* is any of the first three letters of English alphabet}.

We shall use the following symbols in sets:

$\in$ – belongs to
$\notin$ – does not belong to
$\subseteq$ – subset, e.g. $A \subseteq B$
$\supseteq$ – super set, e.g. $B \supseteq A$
N – set of natural numbers
= {1, 2, 3, 4, 5, ...}
W – set of whole numbers
= {0, 1, 2, 3, ...}
Z – set of integers
= {....., –3, –2, –1, 0, 1, 2, 3, ...}
Q – set of rational numbers
$= \{\frac{a}{b}; a, b \in Z, b \neq 0\}$
R – set of real numbers
= {Rational Numbers}
$\cup$ {Irrational Numbers}
$\forall$ – for all
$\exists$ – there exists
$\cup$ – union
$\cap$ – intersection
V – or
$\vee$ – and

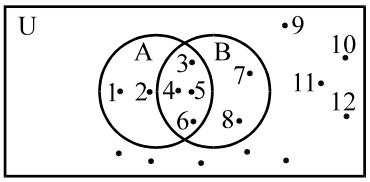
VENN DIAGRAMS

U–UNIVERSAL SET

Definition: If every set under consideration is a subset of a set U, then U is universal set.

e.g. Set : U = N, the set of natural numbers

A = {1, 2, 3, 4, 5, 6}, B = {3, 4, 5, 6, 7, 8}



INTEXT QUESTIONS

Q. 1. From the collections mentioned below, identify the sets from the mathematician's point of view:

(a) **The collection of all good people in India.**

(b) **The collection of all those people who have been to Mars.**

(c) **The collection of prime numbers.**

(d) **The collection of all even natural numbers.**

Solution:

(a) is not a set as goodness is not measurable

(b) is a set

(c) is a set

(d) is a set

Q. 2. Which of the following statements are true?

(a) **$2 \in N$**

(b) **$2 \notin N$**

(c) **$\sqrt{2} \in R$**

(d) **$\sqrt{2} \in Q$**

Solution:

(a) $2 \in N$, True, as 2 is a natural number.

(b) $2 \notin N$, False, 2 being natural, $2 \notin N$ is false.

(c) $\sqrt{2} \in R$, True, $\sqrt{2}$ being irrational is real too.

(d) $\sqrt{2} \in Q$, False, $\sqrt{2}$ being irrational is not rational.

Q. 3. Describe the following sets by the listing method.

(a) **$\{x : x$ is the smallest prime number$\}$**

(b) **$\{x : x$ is a divisor or 12$\}$**

(c) **$\{x : x \in Z$ and $x^2 = 4\}$**

(d) **$\{x : 3x - 5 = 19\}$**

Solution:

(a) $\{x : x$ is the smallest prime number$\}$
$= \{2\}$

(b) $\{x : x$ is a divisor of 12$\}$
$= \{1, 2, 3, 4, 6, 12\}$

(c) $\{x : x \in z$ and $x^2 = 4\}$
$= \{x : x \in z$ and $x = \neq 2\}$
$= \{2, -2\}$

(d) $\{x : 3x - 5 = 19\}$
$= \{x : 3x = 19 + 5\}$
$= \{x : x = \frac{24}{3}\}$
$= \{8\}$

(e) The set of all letters of malayalam
$= \{m, a, l, y\}$

Q. 4. Describe the following sets by the property method.

(a) **{1, 4, 9, 16,.....}**

(b) **{2, 3, 5, 7, 11, 13, 17,.....}**

(c) **{....., – 6, – 4, – 2, 0, 2, 4, 6,.....}**

Solution:

(a) $\{1, 4, 9, 16, 25, ...\}$
$= \{1^2, 2^2, 3^2, 4^2, 5^2, ...\}$
$= \{x : x = y^2, y \in N\}$

(b) $\{2, 3, 5, 7, 11, 13, 17, ...\}$
$= (x : x$ is a prime number)

(c) $\{....., -6, -4, -2, 0, 2, 4, 6, ...\}$
$= \{x : x = 2y, y \in Z\}$

Q. 5. Give an example of a non-empty set which can be represented only by the property method.

Solution:

$\{x \in R : 1 < x < 2\}$

Q. 6. Which of the following sets are finite, and which are infinite?

(a) **Z,**

(b) **ϕ,**

(c) **The set of points on the circumference of a circle,**

(d) **$]0, 1[$,**

(e) **$[-1, 1]$,**

(f) **{1, 2,....., 100}**

Solution:

(a) $Z = \{....., -3, -2, -1, 0, 1, 2, 3, ...\}$ is an infinite set.

(b) ϕ has no element, i.e., number of elements in ϕ is $0 \Rightarrow \phi$ is a finite set.

(c) The set of points on the circumference of a circle is an infinite set.

(d) $]0, 1[$ is an open set of real numbers and is an infinite set.

(e) Close set $[-1, 1]$ of real numbers is an infinite set.

(f) {1, 2, 3, ..., 100} a subset of N has 100 elements, and therefore is a finite set.

Q. 7. Write down all the subsets of {1, 2, 3}. How many of these contain:

(a) **no element,**

(b) **one element,**

(c) **two elements,**

(d) **three elements?**

Solution:

Subsets of {1, 2, 3} are
ϕ, {1}, {2}, {1, 2}, {2, 3}, {1, 3}, {1, 2, 3}

(a) ϕ has no element, i.e. 1 set has no elements or has 0 element.

(b) {1}, {2}, {3} are sets of one element each
$\therefore$ 3 sets have one element.

(c) {1, 2}, {2, 3} {1, 3} have two elements each
$\therefore$ 3 sets have 2 elements.

(d) {1, 2, 3} has 3 elements
$\therefore$ 1 set has 3 elements

Q. 8. Show that if A $\subseteq$ B and B $\subseteq$ C, then A $\subseteq$ C.

(Hint: Show that $a \in A \Rightarrow a \in C$.)

Solution:

To show that if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$

Let $x \in A$

$\Rightarrow$ $x \in B$, as $A \subseteq B$

$\Rightarrow$ $x \in C$, as $B \subseteq C$

$\therefore$ $x \in A \Rightarrow x \in C$

$\therefore$ $A \subseteq C$

Hence, proved.

Q. 9. If A = B and B $\supseteq$ C, what is the relationship between A and C?

Solution:

Given $B \supseteq C$

$\Rightarrow$ $C \subseteq B$

But $A = B$

$\therefore$ $C \subseteq A$

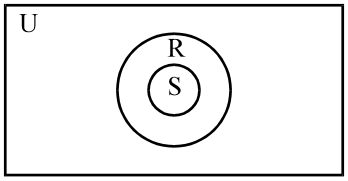
Hence the result.

Q. 10. How would you represent the following situation by a Venn diagram?

The set of all rectangles, the set of all squares and the set of all parallelograms.

Solution:

As a square is in general rectangle and a rectangle is, in general, a parallelogram too, we take universal set U as the set of parallelogram. If S denotes the set of squares and R, the set of rectangles, then the given diagram is as follows:



Q. 11. ***(a)*** **Represent the following sets in a Venn diagram: The set P of all prime numbers, the set Z and the set Q ~ Z.**

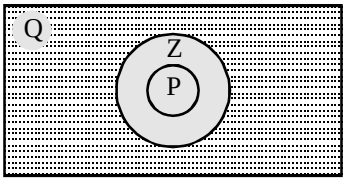
(b) **Is the set Z ~ P finite or infinite?**

Solution:

P = set of all the prime numbers

Z = set of integers

Universal set = Q



$P \subset Z \subset Q$

(a) Q – Z 

(b) Z – P is an infinite set.

Q. 12. If U = {1, 2, 3,.....}, A = {1, 3, 5,.....}, B = {2, 4, 6,.....}, C = {2, 3} and D = {1}, find A $\cap$ B, B $\cap$ C, C $\cap$ D and B $\cap$ D.

Solution:

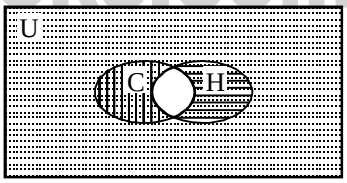
u = {1, 2, 3, ...}, A = {1, 3, 5, ...},
B = {2, 4, 6, ...}
C = {2, 3}, D = {1}
$A \cap B = \phi$
$B \cap C = \{2\}$
$C \cap D = \phi$
$B \cap D = \phi$

Q. 13. Let C and H be the set of carnivorous and herbivorous animal species. Take U to be set of all animals species. Represent H, C, C′, C $\sqcup$ H and H $\sqcup$ C in a Venn diagram.

(Note that $C \cap H \neq \phi$.)

Solution:

C = {Carnivorous}
H = {Herbivorous}
U = {All animal species}



H – C = 

C – H = 

C′ =  + 

Q. 14. Let A = {1, 2, 3, 4}, B = {3, 4, 5, 6} and C = {1, 4, 7, 8}. Determine A $\cap$ B $\cap$ C. Also verify that

(a) **A $\cap$ B $\cap$ C = (A $\cap$ B) $\cap$ C**

(b) **A $\cap$ B $\cap$ C = A (B $\cap$ C)**

Solution:

A = {1, 2, 3, 4}, B = {3, 4, 5, 6},
C = {1, 4, 7, 8}

$A \cap B \cap C = \{4\}$

(a) $A \cap B = \{3, 4\}$

$\therefore \quad (A \cap B) \cap C = \{4\}$

$\therefore \quad A \cap B \cap C = (A \cap B) \cap C$

Verified.

(b) $B \cap C = \{4\}$

$\therefore \quad A \cap (B \cap C) = \{4\}$

$\therefore \quad A \cap B \cap C = \{A \cap (B \cap C)\}$

Verified.

Q. 15. Show that, for any two sets A and B, $A \cap B = B \cap A$, that is, the operation of union is commutative.

Solution:

To show that operation of union is commutative.

Proof: We shall show that

$A \cup B \subseteq B \cup A$

and $B \cup A \subseteq A \cup B$

Let $x \in A \cup B$

$\Leftrightarrow \quad x \in A$ or $x \in B$ or $x \in A \cap B$

$\Leftrightarrow \quad x \in B$ or $x \in A$ or $x \in B \cap A$

$\Leftrightarrow \quad x \in B \cup A$

$\therefore \quad A \cup B = B \cup A$

Hence, proved.

Q. 16. If A is the set of red-haired people and B the set of black-haired people, determine $A \cap B$ and $A \cap B$.

Solution:

A = {Red haired people}

B = {Black haired people}

$\therefore \quad A \cup B$ = {People who have either red hair or black hair}

and $A \cap B = \{\ \} = \phi$

Q. 17. Let U denotes the set of all human beings. A the set of individuals suffering from typhoid and B the set of individuals suffering from cholera. Interpret $A \cup B$.

Solution:

U = {All human beings}

A = {People suffering from typhoid}

B = {People suffering from cholera}

$\therefore \quad A \cup B$ = {People suffering either from typhoid or cholera or both}

Q. 18. Let U be the whole real line R, $A = \{x \in R: 0 \leq x \leq\}$ and $B = \{x \in R : 1 \leq x < 3\}$. Determine $A \cup B$.

Solution:

U = Real number line

$A = \{x \in R;\ 0 \leq x \leq 1\}$

$B = \{x \in R;\ 1 \leq x < 3\}$

$\therefore \quad A \cup B = \{x \in R;\ 0 \leq x < 3\}$

Q. 19. What can you say about A and B if $A \cup B = \phi$?

Solution:

If $A \cup B = \phi$, then

$A = \phi$ and $B = \phi$

Q. 20. If A = {1, 2, 3}, B = {2, 3, 4, 5}, C = {1}, determine $A \cup B \cup C$. Verify that $A \cup B \cup C = (A \cup B) \cup C = A (B \cup C)$.

Solution:

A = {1, 2, 3}, B = {2, 3, 4, 5}, C = {1}

$A \cup B \cup C = \{1, 2, 3, 4, 5\}$ ... (1)

$A \cup B = \{1, 2, 3, 4, 5\}$

$\therefore \quad (A \cup B) \cup C) = \{1, 2, 3, 4, 5\}$ ... (2)

Next $B \cup C = \{1, 2, 3, 4, 5\}$

$\therefore \quad A \cup (B \cup C = \{1, 2, 3, 4, 5\}$ ... (3)

By (1), (2), (3),

$A \cup B \cup C = (A \cup B) \cup C = A \cup (B \cup C)$

Verified.

Q. 21. Let A = {1, 2, 3} and B = {3, 4, 5, 6}, Verify that $\cup \ B = (A \sqcup B) \cup (A \cup B) \cup (B \sqcup A)$.

Solution:

A = {1, 2, 3}, B = {3, 4, 5, 6}

$\therefore \quad A \cup B = \{1, 2, 3, 4, 5, 6\}$ ... (1)

$A \sim B = \{1, 2\}$

$A \cap B = \{3\}$

$B \sim A = \{4, 5, 6\}$

$\therefore \quad (A \sim B) \cup (A \cap B) \cup (B \sim A)$

$= \{1, 2, 3, 4, 5, 6\}$ ... (2)

$\therefore$ By (1) and (2)

$A \cup B = (A \sim B) \cup (A \cap B) \cup (B \sim A)$

Hence, verified.

Q. 22. Let A = {1}, B = {2, 3, 4}, C = {3, 4, 5}. Verify that

(a) $\mathbf{A \cup (B \cup C) = (A \cup B) (A \cup C)}$

(b) $\mathbf{A (\cup\ B C) = (A \cup B) \cap (A \cup C)}$

Solution:

A = {1}, B = {2, 3, 4}, C = {3, 4, 5}

(a) $A \cap B = \{\ \} = \phi$

$B \cup C = \{2, 3, 4, 5\}$

$A \cap C = \{\ \} = \phi$

$\therefore \quad A \cap (B \cup C) = \phi$ ... (1)

Next $(A \cap B) \cup (A \cap C)$

$= \phi \cup \phi$

$\therefore \quad (A \cap B) \cup (A \cap C) = \phi$ ... (2)

$\therefore$ By (1) and (2),

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Hence, verified.

(b) $B \cap C = \{3, 4\}$

$\therefore \quad A \cup (B \cap C) = \{1, 3, 4\}$ ... (1)

and, $A \cup B = \{1, 2, 3, 4\}$

$A \cup C = \{1, 3, 4, 5\}$