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Subhash G. Deo **B.E, P.G.D. (O.R. & S.Q.C.) B.J.**

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QUESTION PAPER

(June – 2018)

(Solved)

ELEMENTARY ALGEBRA

Time: 1½ Hours] *[Maximum Marks: 25*

Weightage: 70%

Note: *Attempt any three questions from questions no. 1 to 4. Question no. 5 is* ***compulsory.*** *Use of caculators is not allowed.*

Q. 1. *(a) (i)* Apply Gaussian elimination method to solve the following systems of equation:

$$-5x + 7y + 3z = 4$$
$$6x + 3y = 8$$
$$3x - 2y = 2 - z$$

Ans.

$$-5x+7y+3z = 4 \quad -(1)$$
$$6x+3y = 8 \quad -(2)$$
$$3x-2y = 2-z \quad -(3)$$

On multiplying by 3 in eq. 3 we get

$$9x-6y-3z = 6 \quad -(4)$$

Substract eq. (4) from eqution (1)

$$-14x+y = -2$$
$$y+2 = 14x$$
$$\frac{y+2}{14} = x \quad -(5)$$

Putting the value of x from equation. (5) in equation (2) we get.

$$6\left(\frac{y+2}{14}\right)+3y = 8$$
$$\frac{6y+12}{14}+3y = 8$$
$$\frac{6y}{14}+\frac{12}{14}+3y = 8$$
$$\frac{6y+42y}{14} = 8-\frac{12}{14}$$
$$\frac{48y}{14} = \frac{112-12}{14}$$
$$48y = 100$$
$$\boxed{y=\frac{100}{48}}$$

Putting the value of y in eqution (5)

$$x = \frac{\frac{100}{48}+2}{14}$$
$$x = \frac{\frac{100+96}{48}}{14}$$
$$\boxed{x=\frac{196}{48\times 14}=\frac{14}{48}}$$

$$3\left(\frac{14}{48}\right)-2\left(\frac{100}{48}\right)+z = 2$$
$$\frac{42}{48}-\frac{200}{48}+z = 2$$
$$\frac{42-200}{48}+z = 2$$
$$\frac{-158}{48}+z = 2$$
$$\boxed{z=2+\frac{158}{48}=\frac{254}{48}}.$$

(ii) **On the basis of the solution you have obtained in *(i)* above could you have used the Cramer's rule to obtain the solution? Give reasons for your answer.**

Ans.

$$\begin{aligned} -5x+7y+3z &= 4 \\ 6x+3y &= 8 \\ 3x-2y &= 2-z \end{aligned}$$

By Cramer's rule

$$D = \begin{bmatrix} -5 & 7 & 3 \\ 6 & 3 & 0 \\ 3 & -2 & 1 \end{bmatrix}$$

$D = -5\,[3-0]-7\,[6-0]+3\,[-12-9]$

$D = -15-42-63$

$D = -120 \neq 0$

So it can be solved by Cramer's rule

$$D_1 = \begin{bmatrix} 4 & 7 & 3 \\ 8 & 3 & 0 \\ 2 & -2 & 1 \end{bmatrix}$$

$D_1 = 4\,[3-0]-7\,[8-0]+3\,[-16-6]$

$= 12-56-66$

$D_1 = -110$

$$D_2 = \begin{bmatrix} -5 & 4 & 3 \\ 6 & 8 & 0 \\ 3 & 2 & 1 \end{bmatrix}$$

$= 5\,[8-0]-4\,[6-0]+3\,[12-24]$

$= -40-24-36$

$= -100$

$$D_3 = \begin{bmatrix} -5 & 4 & 3 \\ 6 & 8 & 8 \\ 3 & -2 & 2 \end{bmatrix}$$

$= 5\,[6+16]-7\,[12-24]+4\,[-12-9]$

$= -110+84-84$

$= -110$

$$x = \frac{D_1}{D} = \frac{110}{120},\ y = \frac{D_2}{D} = \frac{110}{120},\ z = \frac{110}{120}$$

Reason: We solve the equation using Cramer's Rule but both solution did not match.

***(b)* For sets A, B and C in a universal set U, show that**

$A \times (B \setminus C) = (A \times B) \setminus (A \times C)$.

Ans. $A \times (B \setminus C) = (A \times B) \setminus (A \times C)$

or $(A \times (B-C)) = (A \times B) - (A \times C)$

let $(x, y) \in (A \times (B-C))$

$x \in A$ and $y \in (B-C)$

$x \in A$ and $(y \in B$ and $y \in C)$

$(x \in A$ and $y \in B)$ and $x \in A$ and $y \in C$

$(x, y) \in (A \times B)$ and $(x, y) \in (A \times C)$

$(x, y) \in (A \times B) - (A \times C)$

$A \times (B-C) \leq (A \times B) - (B \times C)$ —(1)

Conversely

Let $(x, y) \in (A \times B) - (A \times C)$

$(x, y) \in (A \times B)$ and

$(x, y) \not\in (A \times C)$

$(x \in A$ and $y \in B)$ and

$(x \in A$ and $y \in C)$

$\Rightarrow x \in A$ and $(y \in B)$ and $y \not\in C)$

$\Rightarrow x \in A$ and $y \in (B-C)$

$\Rightarrow (x, y) \in A \times (B-C)$

$\Rightarrow (A \times B) - (A \times C) \leq A \times (B-C)$ —(2)

from (1) & (2)

$A \times (B-C) = (A \times B) - (A \times C)$.

Q. 2. *(a)* Solve the equation:

$54x^3 - 39x^2 - 26x + 16 = 0$,

Given that the roots are all real and are in G.P.

Ans. $54x^3 - 39x^2 - 26x + 16 = 0$ ...(1)

As the roots are in C.P. we may take them as $\frac{\alpha}{i}$, α, αr.

$$\text{Now, } \frac{\alpha}{r} + \alpha + \alpha r = +\frac{39}{54} \qquad -(2)$$

$$\left(\frac{\alpha}{r}\right)(\alpha) + \alpha\,(\alpha r) + \left(\frac{\alpha}{r}\right)(\alpha r) = +\frac{26}{54} \qquad -(3)$$

$$\left(\frac{\alpha}{r}\right)(\alpha)\,(\alpha r) = \frac{16}{54} \qquad -(4)$$

From (4), we get $\alpha^3 = \frac{16}{54} = \frac{8}{27} \Rightarrow \alpha = \frac{2}{3}$

Putting this in (2), we get

$$= \frac{2}{3}\left(\frac{1}{r}+r\right) = \frac{39}{54} - \frac{2}{3}$$

$$\text{or} \quad \frac{2}{3}\left(\frac{1}{r}+r\right) = \frac{13}{18} - \frac{2}{3} \Rightarrow \frac{13-12}{18}$$

$$\frac{2}{3}\left(\frac{1}{r}+r\right) = \frac{1}{18}$$

Sample Preview
of
The Chapter

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ELEMENTARY ALGEBRA

SOLUTIONS OF POLYNOMIAL EQUATIONS

Sets

1

INTRODUCTION

In mathematics one of the basic concepts is that of **numbers**, *i.e.* 0, 1, 2, 3, ... to infinity. These numbers form the foundation of all mathematical operation. A similar basic concept is that of **Sets**. This concept was developed only in the last century, but is already at the core of all mathematical operations.

A dictionary contains words.

The collection of all words in a given dictionary can be called a **Set**. A word will belong to this set depending upon whether it is listed in the dictionary or not.

In this chapter, we will study the basic ideas and properties related to sets.

Our study will consist of following activities:

1. Definition of a set.
2. How to identify a set?
3. Representing sets by:
   (i) Listing method
   (ii) Property method, and
   (iii) Venn diagram.
4. Proving and using the Laws of Distribution.
5. Proving and using DeMorgan's Laws.
6. Definition of Cartesian Product of sets, and how to get it for two or more sets.

What is a Set?

In our daily routine, we always come across the groups, classes, categories or collections of objects. For example, a group of friends, a category of detective books, a collection of Mukesh songs, or a class of all Cars, and so on.

In mathematics, a **well-defined** collection of objects is called a **set**. Here **well-defined** means when an object is given we should be able to clearly tell if it belongs to the collection of objects or not. For example, the collection of all women scientists is a set. A given person will belong to this set (collection) only if the person is a woman and also a scientist. These two properties are clearly (well) defined.

However, the collection of nice persons is not a set. This is because a particular person may be nice according to some people, and may not be nice according to other people. Clearly, we have no criteria to decide that who is nice, and who is not. Thus, this collection is not well-defined and cannot be called a set.

Examples of Sets in Mathematics:

1. Set of integers, denoted by Z.
2. Set of real numbers, denoted by R.
3. Set of rational numbers, denoted by Q.
4. Set of natural numbers, denoted by N.

5. Set of complex numbers, denoted by C.

What is a Prime Number?

A prime number is a natural number other than one, whose only factors are one and itself e.g. 3, 7, 2, 43, 17, 191...

Note that:

1. The number 1 is not a prime number.
2. The lowest prime number is 2.
3. The only even prime number is 2, all others are odd.

Element or Member of a Set: An object that belongs to a set is called an element or member of that set. Thus, 7 is a member of the set of all odd natural numbers.

These elements of sets are always denoted by small letters of alphabet, *i.e.*, *a, b, c,* ....., *x, y, z*.

The sets are usually denoted by capital letters, A, B, C, ..., X, Y, Z.

The symbol $\in$ (pronounced *epsilon*) is used to mean 'belongs to'. Italian mathematician Peano (1858-1932) was the first to suggest and use it in practice.

Using $\in$, we can symbolically write the statement '$\underline{a}$ is an element of the set A' as $a \in$ A.

On the other hand, if $\underline{a}$ is not an element of A *i.e.*, 'if $\underline{a}$ does not belong to A', we write this statement as $a \notin$ A.

Here, $\notin$ reads 'does not belong to.'

As a further example, if P is the set of prime numbers, then $11 \in$ P, and $12 \notin$ P.

Null or Empty Set (ϕ): A person can be either married or unmarried, but not both at the same time. Similarly, a number can be either rational or irrational, but not both.

Therefore, if a set of all numbers that are rational as well as irrational, is formed, it will have no element at all.

Such a set, which has no elements, is called an **empty set** or **void set** or **null set**. It is denoted by the Greek letter ϕ (pronounced *phi*).

Non-empty Set: A set which has at least one element is called a non-empty set. A non-empty set can be described in two ways:

(a) The Listing Method.

(b) The Property Method.

(a) **The Listing Method (Also called Roster Method or Tabular Method):** It this method all the elements of the set are listed within curly brackets. For example, set of all natural numbers that are factors of 12 is {1, 2, 3, 4, 6, 12}.

If a set consists of a large number of elements, the initial few elements of the set are listed to make the pattern of the set clear, followed by ....

For example, the set N of natural numbers can be written as:

N = {1, 2, 3, ...}

Similarly, the set of all numbers that can be divided by 7 between 20 to 100 can be written as:

{21, 28, 35, ..., 98}

This method of representing sets is called as **Listing,** or **Roster,** or **Tabular Method**.

(b) **Property Method or Set-Builder Method:** The second method of representing a set is called as **Property** or **Set-builder Method.** In this, the members of a set are described using a common property (feature), which they all have.

For example, suppose we want to represent a set of all natural numbers which are multiples of 10. This set S can be written as:

S = {x | $x \in$ N and x is a multiple of 10 } .... (I)

Here, x is any element of the set. The vertical bar after x reads 'such that'.

Thus, the representation (I) states that 'S is the set of all x' such that x is a natural number and x is a multiple of 10.

Two shorter ways of writing above statement are:

S = {$x \in$ N | x is a multiple of 10}

S = {10n | $n \in$ N}.

Use of the Colon: is also allowed in place of the vertical bar 1.

Thus, we can write,

S = {$x \in$ N : x is a multiple of 10}

or

S = {10$n \in$ N : $n \in$ N}

Obviously, in some cases we can describe the set using either the listing method or the property method.

For example, a set D, of all natural numbers less than 6, can be expressed as either

D = {1, 2, 3, 4, 5} by the listing method, or as

D = {x | x is a natural number less than 6}

by the property method.

Clearly, both the sets are same with exactly same elements in them. In other words, there are equal or equivalent sets.

Definition of Equal Sets: Two sets **S** and **T** are said to be equal, denotes as **S = T**, if and only if every element of S is an element of T and every element of T is an element of S.

Points to Remember about Listing Method:

(i) The set {1, 2, 3} is same as the set {1, 2, 3}. In a set an element is not repeated. Any element is listed only once.

(ii) Changing the order in which the elements are listed does not alter (change) the set. Thus, the sets {1, 2, 3} and {2, 1, 3} and {3, 2, 1} are equal. This is because every element of any of these sets is present in the two other sets.

Point to Remember about the Property Method: Note that there can be several properties that can define the same set.

As examples, consider

(a) {x | $4x - 1 = 7$}
= {x | x is an even prime number}

(b) = {x | x is divisible by 2}
{x | x is an even number}.

Singleton, finite and infinite sets: A set consisting of exactly one element is called a singleton set. Such a set with element x is denoted as {x}.

Note that the element x is not the same as the set {x}. Both are different. Clearly, we can say that, $x \in \{x\}$.

A set which has a finite number of elements is called a **finite set**. The empty set is also defined as a finite set.

A set which is not finite, is an infinite set. Examples of an infinite set are: N, Q, R the set of points on a given line, etc.

Subsets of a Set: A set can consist of two or more small subset contained within it.

For example, let A and B be two sets as defined below:

A = The set of all people in the world, and
B = The set of all women in the world.

Every woman in the world clearly also belongs to people of the world. So, each element of B is also an element of A. In such conditions, it is said that B is contained in A.

However, in the set A, people of the world contain men also apart from women.

Therefore, there are one or more elements in A which also do not belong to B. We express this fact as 'there is an element x in A such that x does not belong to B.'

In mathematical ways, we write this as:

$\exists\ x \in$ A such that $x \notin$ B.

This is read as: "for all x belonging to A, such that x does not belong to B."

Here, we can then state that B is properly contained in A.

The Symbol ∃
Symbol $\exists$ denotes 'there exists'. Thus $\exists\, x$ means 'there exists x'.

Definition of a Subset: A set A is a subset of set B if every element of A belongs to B. Symbolically, this is written as $A \subseteq B$.

This is read as 'A is contained in B.'

The same fact can be written as $B \supseteq A$.

This is read as B contains A.

Definition of Proper Subset: If $A \subseteq B$ and $\exists y \in$ B such that $y \notin$ A, then A is called a proper subset of B or it is said that 'A is properly contained in B.' This fact is symbolically denoted by:

$A \subseteq B$.

On the other hand, if X and Y are two sets such that X has an element x which does not belong to Y, then it is said that X is not contained in Y. This is written as:

$X \notin Y$.

Some Properties of Subsets:

1. Any set is a Subset of itself: This means $A \subseteq A$, or $B \subseteq B$.

However, no set is a proper subset of itself. is y definition of subset, this is not possible.

2. The empty set ϕ is a subset of every set: Let there be two sets A and B, such that

A = {5, 7} and B = {3, 7, 10}

Therefore, $\exists$ (5) $\in$ A such that (5) $\notin$ B.

$\therefore\quad A \not\subseteq B$

Similarly $B \not\subseteq A$

Now, clearly for any given two sets A and B, one and only one of the following two possibilities can be true:

Either $A \subseteq B$

or $A \not\subseteq B$

Using these two facts, it can be shown by contradiction that the empty set ϕ is a subset of every set.

i.e., $\phi \subseteq A$ for any set A.

Proof: To prove that the empty set ϕ is a subset of every set, let us take any set A.

Suppose $\phi \not\subseteq A$.

Then there has to be an element in ϕ which is not in A. However, this is just not possible because ϕ has no elements. Thus, there is a contradiction and our starting assumption is wrong. This means $\phi \not\subseteq A$ is false.

And, what must be true is $\phi \subseteq A$,

For any set A.

Use of Symbol $\Rightarrow$ (reads 'implies')

The symbol $\Rightarrow$ means 'implies'.

For example, say

$x \in A \Rightarrow x \in B$ read as,

x belongs to A implies that x belongs to B.

Also, $x \in A \not\Rightarrow x \in B$ read as.

x belong to A does not imply that x belongs to B.

Definition of Power Set: A power set is defined as the set of all subsets of a set.

Thus, if A = {0, 1} its

Power set A^p is given by $\{\phi\}$,

$$A^p = \{\{0\}, \{1\}, \{0, 1\}\}$$

Also if B = {0, 1, 2}, its power set B^p is given by:

$$B^p = \{\{0\}, \{1\}, \{2\}, \{0, 1\}, \{0, 2\}, \{1, 2\}, \{0, 1, 2\}\}$$

Remember that the empty set $\{\phi\}$ is always a subset of any set.

Equality of Subsets: Equality of two sets A and B means every member of set A is a member of set B and *vice versa*. i.e., $A \subseteq B$ and $B \subseteq A$.

Symbolically putting we have,

$$A = B \Leftrightarrow (A \subseteq B \text{ and } B \subseteq A)$$

As an example, let

A = The set of odd numbers less than 9, and

B = {1, 3, 5, 7}

A and B are equal since every member of A is a member of B, and *vice versa* i.e. A, $\supseteq$ B and $B \subseteq A$.

Therefore,

$$A = B \Leftrightarrow (A \subseteq B \text{ and } B \subseteq A).$$

Venn Diagram: Apart from the Listing (Roster) Method and the Property Method, the third method of representing sets is the Venn Diagrams. These diagrams very clearly depict sets and the relationship among them.

Because Venn diagrams show us the sets details and their relationships graphically, it becomes easier to understand the situation correctly.

An English Mathematician John Venn (1834-1923) invented this method. That is why the diagrams are named after him.

For the use of Venn diagram, John Venn also introduced the concept of universal set.

Definition of Universal Set: An universal set is a set which is large enough and is convenient to contain all the sets under study. Such a large set is called as a **universal set** and is denoted by the capital letter U.

An important fact about the universal set is, it is not unique. There can be more than one universal set which can contain the various sets under discussion.

For example, suppose there is the set of girl chess players and also the set of girl tennis players. Then we can have the universal set U of all girl players. This is because U contains both types of girls. However, we can take the universal set as that of all sports players. That will also serve the same purpose.

Another example can be cited of the set of **rational numbers** and another set of **integers**. For these two sets, their universal set can be the set of **real numbers R,** or it can be the set **rational numbers Q**, since both contain Z and Q.

In general, a universal set is chosen which is just large enough to contain all the sets being studied.

How to draw a Venn Diagram?

Let there be various sets A, B, C, ... under our study. Also, let the universal set be U. Then, $A \subseteq U$, $B \subseteq U$, $C \subseteq U$ and so on. This situation is shown on a Venn diagram in following way:

A rectangle is drawn to represent the universal set U. In side the rectangle U, the subsets A, B, C, ... etc. are represented in the shapes of closed curves like circle, ellipse or any other shape. If any of the subsets have common elements, they intersect each other and their