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CONTENTS

ABSTRACT ALGEBRA

■■

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QUESTION PAPER

(June-2016)

(Solved)

ABSTRACT ALGEBRA

Time: 2 hours] *[Maximum Marks: 50]*

[Weightage : 70

Note: *Answer any **five** questions. All questions carry **equal** marks.*

Q. 1. (*a*) Define a relation R on the set of integers *Z* by R = {(*n, n* + 3*k*)} | *k* ∈ *Z*. Show that R is an equivalence relation. Also find all distinct equivalence classes.

Sol. R = {(*n, n* + 3*k*) | *k* ∈ *Z*}

Since R is an equivalence relation it is reflexive.

Thus $k\text{R}k \ \forall \ k \in Z$, then $k \in [k]$

Firstly assume that $n \in [k]$ we will show that $[n] \subseteq [k]$ and $[k] \leq [n]$. For this $x \in [k]$, then *x*R*k*. We also know that *a*R*b*. by using transivity of R. i.e. $x \in [k]$. Thus $[n] \subseteq [k]$.Similarly show that $[k] \subseteq [n]$. Thus [*n*] = [*k*].

So R is an equivalence relation on the set of integer *Z*.

(*b*) Express the permutation

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 8 & 6 & 7 & 4 & 1 & 5 & 2 \end{pmatrix}$$ **first as a product of disjoint cycles and then as a product of transpositions. What is the signature off?**

Sol. $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 8 & 6 & 7 & 4 & 1 & 5 & 2 \end{pmatrix}$

= (1, 1) (2, 2) (4, 4) (5, 5)

The signature of F.

Sign $f = \prod_{m_{ij}} = 1$

$1 < j\{\text{I}(i) - \text{I}(i)\}|(j-1) = 1$

(*c*) Show that the polynomial

$$\mathbf{3x^5 + 15x^4 - 20x^3 + 10x + 20}$$

is irreducible over *Q*. Is the polynomial $3x^2 + x + 4$ irreducible over Z_7? Give reasons.

Sol. $3x^5 + 15x^4 - 20x^3 + 10x + 20$

Reducing the coefficients modulo 3 gives the polynomial $x^5 + x^4 + x + 20$, which is irreducible in $Z_3(x)$. This implies that the polynomial is irreducible over Q.

If P(*x*) = $3x^2 + x + 4$, then

P(0) = 4, P(1) = 8, P (–1) = 6

P(2) = 18 and P(–2) = 14, P(3) = 34, P(–3) = 28

So P(*x*) is irreducible over Z_7.

Q. 2. (*a*) Find the remainder of 37^{49} when divided by 7.

Sol. Using Fermat's theorem, for *x* not divisible by P, $x^{(P-1)} = 1$ mod *p* or $x^{\wedge P} = x$ mod *p*

So 37^{49} = 37 mod 49.

So there is 9 such that

$37^{49} = 49q + 37$

Dividing by 7 results in 7*q* + 5 with a remainder of 2.

(*b*) Let S be the set of all real numbers except –1. Define an opration ⊕ on S by *x* ⊕ *y* = *x* + *y* + *xy*; *x, y* ∈ S. Show that < S, ⊕ > is an abelian group. Find a solution of the equation 1 ⊕ *x* = 2 in S.

Sol. As we know that ⊕ is well defined binary operation on S.

$$\begin{aligned} x \oplus y &= x + y + xy \\ &= y + x + 1 \\ &= y \oplus a \end{aligned}$$

$$\begin{aligned}(x\oplus y)\oplus 2 &= (x+y=1)\oplus z\\ &= x+y+1+z+1\\ &= x+(y+z+1)+1\\ &= x\oplus(y\oplus z)\end{aligned}$$

$$\begin{aligned}1\oplus x &= 1+x+1\\ &= 1+x\,.\,x^{-1}+1\\ &= 2+0\end{aligned}$$

$$\boxed{1\oplus x = 2}$$

(*c*) Find the nil radical of Z_8.

Sol. Let the nil radical of Z_8 be N.

Then $\bar{0}$ N

$\bar{1} \notin$ N since $\bar{1}n = \bar{1} \neq \bar{0}$ for all n

$\bar{2}^3 = \bar{0} \Rightarrow \bar{2} \in$ N

$\bar{3}_n \neq \bar{0}\ \forall\ n$

Therefore $\bar{3} \in$ N

Similarly $\bar{4}, \bar{6} \in$ N and $\bar{5}, \bar{7} \notin$ N

So that N = $\{\bar{0}, \bar{2}, \bar{4}, \bar{6}\}$

For any A $\notin \phi$ (X), $A^n = A \cap A \cap ... = A\ \forall\ n$

Therefore $A^n = \phi$ iff $A = \phi$

Thus the nil radical of Z_8.

Q. 3. (*a*) Let R = $Z + \sqrt{2}\,Z$ and S = the ring of 2 × 2 matrices of the form $\left\{\begin{bmatrix} a & 2b \\ b & a\end{bmatrix} a, b \in Z\right\}$. Show that $\theta : R \to S$ defined by $\theta(a+\sqrt{2}b) = \begin{bmatrix} a & 2b \\ b & a\end{bmatrix}$ is an isomorphism of rings.

Sol. R = Z + $\sqrt{2}$ = $\begin{bmatrix} a & 2b \\ b & a\end{bmatrix}$

with Z + $\sqrt{2}$ in R, a, b in Z and the 2 × 2 matrix in R'

$$= \begin{bmatrix} a & 2b \\ b & a\end{bmatrix}\begin{bmatrix} c & 2d \\ d & c\end{bmatrix}$$

To show this map is onto suppose there is an element in R'

$$= \begin{bmatrix} r & 2s \\ s & r\end{bmatrix}$$

Which is not the image of at least one element in R. Then there is no element in R that can be mapped to the element above which is in R′. But there exists integers $r, s,$ in Z such that Z + $\sqrt{2}$

Then since the map is both 1 – 1 and on to it must be an isomorphism from R to R′.

(*b*) If G is a finite commutative group of order *n* and if a prime *p* divides *n*, show that the number of Sylow-*p* subgroups of G is one. Find the unique Sylow-3 and Sylow-2 subgroups of the cyclic group Z_{24}.

Sol. Let G be a finite group of order n and let p be a prime no. All Sylow p-subgroups of G are conjugate and any p-subgroup of G is contained in a Sylow p-subgroup.

Let $n = m\,p^k$ with $gcd\,(m, p) \equiv 1$ and let s be the no of Sylow p-subgroups of G. Then

S | m and S ≡ 1 (mod p)

The no. of 2-Sylow subgroups is either 1 or 3 and there is a unique conjugacy class of such subgroups.

The no. of 3 Sylow subgroups is either 1 or 4 and there is a unique conjugacy class of such subgroups.

(*c*) Is $I = \left\{\begin{bmatrix} m & 0 \\ n & 0\end{bmatrix} m, n \in Z\right\}$ an ideal of the ring $R = \left\{\begin{bmatrix} k & l \\ m & n\end{bmatrix} k, l, m, n \in Z\right\}$ of 2 × 2 matrices over integers? Justify your answer.

Sol. I = $\begin{bmatrix} m & 0 \\ n & 0\end{bmatrix}$: R = $\begin{bmatrix} k & l \\ m & n\end{bmatrix}$

$$IR = \begin{bmatrix} m & 0 \\ n & 0\end{bmatrix}\begin{bmatrix} k & l \\ m & n\end{bmatrix} = \begin{bmatrix} mk & ml \\ nk & nl\end{bmatrix}$$

$$RI = \begin{bmatrix} k & l \\ m & n\end{bmatrix}\begin{bmatrix} m & 0 \\ n & 0\end{bmatrix} = \begin{bmatrix} km + l\text{n} & 0 \\ m^2 + n^2 & 0\end{bmatrix}$$

IR ≠ RI

Hence it is not ideal.

Q. 4. (*a*) Prove by the principle of mathematical induction that 2^n. 3^n –1 is divisible by 17 for all positive integers *n*.

Sol. Let P(n) = 2^n, 3^{2n} –1 is divisible by 17

For n = 1.

Sample Preview
of
The Chapter

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ABSTRACT ALGEBRA

ELEMENTARY GROUP THEORY

Sets and Functions

1

INTRODUCTION

This chapter has the basic concepts related to sets and functions. It is the branch of mathematics, particularly algebra in which we study brief of the elegance of number theory. Here we will study the state and use different types of sets and subsets, about Cartesian product, relation of the elements and state and use of different type of functions.

CHAPTER AT A GLANCE

SETS

Sets are one of the most fundamental concepts in mathematics and which are collections of distinct objects. Although any type of object can be collected into a set.

For example, (a, ñ b),(1,2,3)

The objects that make up a set are called members or elements of the set. The objects that make up asset are called **members** or **elements** of set. An object can be anything that is "meaningful". For example, a number, a person, an equation, another set.

Two sets are equal if they have the same members, that is, a set is completely determined by its members. this is known as the **principle of extension**.

When an object x is an element of some set X, we say x belongs to X and write $x \in$ X. For example, if the set X = {1, 2, 3}, we say that 1, 2, and 3 are the elements of X and write $1 \in$ X, $2 \in$X, and $3 \in$X.

Those sets are **Empty Set** which does not have any elements.

There are two way to describing a non-empty set.

1. Roster Method: The Roster method includes listing the elements within braces.

For example: A set of positive integers less than 6 will be written as

{1,2,3,4,5}

A set of even natural numbers will be written as

{2,4,6….}

In this method two conventions are followed:

1. The order in which the elements are written is not important.

2. No element should be written more than once.

2. Set Builder Method: This method is also known as Property method. In this method, a set is represented by stating all the properties which are satisfied by the elements of the set and not by any other element.

If A contains all values of 'x' for which the condition P(x) is true, then we write A = {x: P(x)}.

Example:

A = {2, 4, 6, 8, 10} write this set in set builder form.

Sol. A = {x/x is even and $x < 12$}.

Subsets: A subset is a proportion of a set. Let B is a subset of A (written $B \subseteq A$) if every member of B is a member of A. If B is a proper subset of A (i.e. a subset other than the set itself), this is written $B \subseteq A$. If B is not a subset of A, this is written $B \not\subseteq A$. (The notation $B \not\subset A$ is generally not used, since $B \not\subseteq A$ automatically means that B and A cannot be the same.

For example, we have the set {1, 2, 3, 4, 5}. A subset of this is {1, 2, 3}. Another subset is {3, 4} or even another, {1}. However, {1, 6} is not a subset, since it contains an element (6) which is not in the parent set. In general: A is a subset of B if and only if every element in A is in B.

Union: If two sets are given, a set can be formed by using all the elements of the two sets. Such a collection is called the union of the given sets.

Union of two sets A and B is the set of all elements which are in A, or in B, or both in A and B. In symbols we write, $A \cup B$ for the union of two sets A and B.

'$A \cup B$' is read as 'A union B' or 'A cup B'. In Set builder form,

$$A \cup B = \{x: X \in A \quad \text{or} \quad X \in B\}$$

Example: Let A = {3, 4, 5, 6}, B = {6, 7, 8} and C = {7,8,9}

$$A \cup B = \{3, 4, 5, 6, 7, 8\}$$
$$B \cup C = \{6, 7, 8, 9\}$$
$$A \cup C = \{3, 4, 5, 6, 7, 8, 9\}$$

Intersection: A set can be formed by using all the common elements of two given sets. Such a collection is called the intersection of the given sets.

Intersection of two sets A and B is a set whose elements belong to both A and B.

In symbols we write, $A \cap B$ for the intersection of two sets A and B.

$A \cap B$ is ready as A intersection B.

In Set-builder form: $A \cap B = \{x: x \in A \text{ and } x \in B\}$

Example: Let A = {3, 4, 5, 6}, B = {5, 6, 7}, C = {7, 8, 9}

Then

$A \cap B = \{5, 6\}$ $\because$ $5 \in A, 5 \in B, 6 \in A, 6 \in B$

$B \cap C = \{7\}$ $\because$ $7 \in B, 7 \in A$

$A \cap C = \phi$ $\because$ no elements are common

Differences: Difference of two sets A and B, A – B is a set whose elements belong to A, but not to B. A – B is called the relative complement of B with respect of A.

Example:

Let A = $\{a, b, c, d\}$, B = $\{c, a, e, f\}$

Then $A - B = \{b, d\}$ $\quad b, d \in A \quad b, d \notin B$

$B - A = \{e, f\}$ $e, f \in B$ $e, f \notin A$

Observe that $(A - B) \cap (B - A) = \phi$

A – B and B – A are disjoint sets.

Complement: Let U be a universal set, A be any subset of U, then the elements of U which are not in A i.e. U – A is the complement of A with respect to U is written as, $\overline{A} = U - A = A^c$.

$$\overline{A} = \{x : x \in U \text{ and } x \notin A\} = A^c$$

Examples:

Let $\quad U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$\quad A = \{1, 2, 4, 6, 9\}$

Then $\quad \overline{A} = U - A = \{0, 3, 5, 7, 8\}$

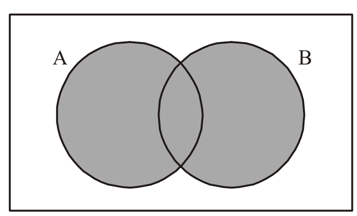
Boolean Operations: Assume now, that all of our sets under discussion are subsets of some larger set E. Now it is easy to make new sets out of given ones through use of the logic operators 'or', 'and', 'for some', 'for all', and 'not' along with specification.

For sets A and B define their union $A \cup B$ as

$\{x \in E \mid x \in A \text{ or } x \in B\}$

Note that $A \subseteq A \cup B$ and $B \subseteq A \cup B$.

Taking the union is a way of 'adding' sets together. You may have seen that the so-called Venn Diagrams illustrating the concept where the shaded region illustrates $A \cup B$.



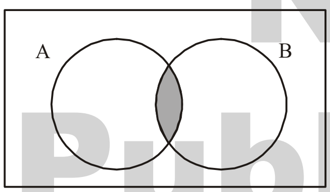
Of course we can take the union of an arbitrary number of sets by replacing the logic operator 'or' with 'for some'. Let C be a collection of subsets of our larger set E, then we define the union of the collection C to be

$\{x \in E \mid x \in X \text{ for some } X \in C\}$ and denote this set by $U_x \in_c X$.

Here are some facts about the union of sets that you should prove for yourself:

1. $A \cup \phi = A$
2. $A \cup B = B \cup A$
3. $A \cup (B \cup C) = (A \cup B) \cup C$
4. $A \cup A = A$
5. $A \subseteq B \Leftrightarrow A \cup B = B$

A second operation on sets is obtained by replacing 'or' and 'for some' in the above discussion with 'and' and 'for all' respectively. Here we can finesse out our larger set E by defining the intersection of sets A and B as the set $\{x \in A \mid x \in B\}$. Denote this new set by $A \cap B$ and note that the intersection $A \cap B$ also equals $\{x \in B \mid x \in A\}$ and $\{x \mid x \in A \text{ and } x \in B\}$. Here is the 'intersection' Venn Diagram where the shaded region represents $A \cap B$.



Here now are the basic facts about intersections,

1. $A \cap \phi = \phi$
2. $A \cap B = B \cap A$
3. $A \cap (B \cap C) = (A \cap B) \cap C$
4. $A \cap A = A$
5. $A \subseteq B \Leftrightarrow A \cap B = A$

Given a non-empty collection C of sets, we can find a set V that contains exactly those elements common to every set in the collection and nothing else. This is because we can find a set A in our collection and define

$V = \{x \in A \mid x \in X \text{ for every } X \in C\}$.

Do not make the mistake of trying to define the intersection of an empty collection of sets. Such an object would be a collection of everything! (For any given object, find me a set in your empty collection that doesn't contain the object) Of course, Russel's Paradox says such a collection could never be a set.

Returning to the intersections of non-empty collections C of sets, we denote such by $\cap_x \in_C X$.

In mathematics, we find collections of sets with empty intersection often enough to give such collections a special name. We call two sets A and B with $A \cap B = \phi$ a disjoint pair of sets. The term is also used occasionally for larger collections of sets with empty intersection, but when any two sets of a collection are disjoint we call the collection pairwise disjoint to indicate this stronger condition.

Example: Produce a collection of three sets with empty intersection which is not pairwise disjoint.

Sol. Two useful formulas involving both unions and intersections are the Distributive Laws:

1. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
2. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

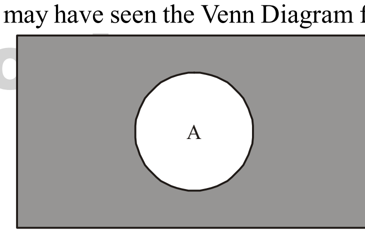
Example: Show that a necessary and sufficient condition for

$(A \cap B) \cup C = A \cap (B \cup C)$ is that $C \subset A$.

Note the condition has nothing to do with the set B.

Sol. Our next formulation of new sets from old involves the logic operator 'not'. In particular for a subset A of our larger set E we consider the complement of A in E as the set $A^c = \{x \in E \mid x \text{ is not an element of } A\}$.

You may have seen the Venn Diagram for this too:



where the shaded region represents A^c. Most of the time our larger set E is clear from the context, so our notation for A^c should be clear. Here are the basic rules about complementation, which again you should prove for yourself

1. $(A^c)^c = A$
2. $\phi^c = E, E^c = \phi$
3. $A \cap A^c = \phi, A \cup A^c = E$
4. $A \subseteq B \Leftrightarrow B^c \subseteq A^c$

But by far the most important statements about complements are the DeMorgan Laws:

5. $(A \cup B)^c = A^c \cap B^c$
6. $(A \cap B)^c = A^c \cup B^c$

Note that these facts about sets indicate that valid formulas involving sets usually come in pairs. If in an inclusion or equation involving unions, intersections and complements of subsets of E we replace each set by its complement, interchange unions and intersections, and reverse inclusions, the result is another valid formula. This is known as the Principle of Duality.

Sometimes we don't care at all about a larger set E and only wish to know about elements of a given set A that are not elements of some other set B. For this, we denote by A – B, the relative complement of B in A which is just the set $\{x \in A \mid x$ is not an element of B$\}$.

It's time now to consider the collection of all subsets of a given set E. The axioms of set theory assert that this collection is again a set, called the power set of E and is denoted by P(E). The set P(E) then is just $\{A \mid A \subseteq E\}$.

Power sets are fairly 'large' when compared to their parent sets. For example, if E = ;, P(E) = $\{\phi\}$; if E = $\{a\}$, then P(E) = $\{\phi, \{a\}\}$; if E = $\{a, b\}$, then P(E) = $\{\phi, \{a\}, \{b\}, \{a,b\}\}$.

In general, if E has n elements, then there are 2^n elements in P(E).

Now if C is a collection of subsets of E, that is, a subset of P(E), then there is another natural collection D of subsets of E, namely $\{X \mid X^c \in C\}$. It is customary to denote the union and intersection of the collection D as $\cup_x \in_C X^c$ and $\cap_x \in_C X^c$. With this notation the generalized DeMorgan Laws become

1. $(\cup_x \in_C X)^c = \cap_x \in_C X^c$
2. $(\cap_x \in_C X)^c = \cup_x \in_C X^c$

CARTESIAN PRODUCT

The Cartesian product of two sets A and B (also called the product set, set direct product, or cross product) is defined to be the set of all points (a, b) where $a \in A$ and $b \in B$. It is denoted $A \times B$, and is called the Cartesian product, named after the French philosopher and mathematician Rene Descartes (1596-1650). He also invented the Cartesian coordinate system. In the Cartesian view, points in the plane are specified by their vertical and horizontal coordinates, with points on a line being specified by just one coordinate.

The main examples of direct products are Euclidean three-space ($\mathbf{R}\times\mathbf{R}\times\mathbf{R}$, where are the real numbers), and the plane ($\mathbf{R}\times\mathbf{R}$).

Example:

If A = {3, 5}, B = {2, 4, 6}, write the Cartesian product *(i)* A × B *(ii)* B × A

A × B = {(3, 2), (3, 4), (3, 6), (5, 2), (5, 4), (5, 6)}

B × A = {(2, 3), (2, 5), (4, 3), (4, 5), (6, 3), (6, 5)}

- **Ordered pair:** Let A= $\{a,b,c\}$ and B= $\{d, e, f\}$. If we take a pair (a,e) then it is being called as an ordered pair as the 1st element of the pair is from set A and the 2nd is from set B.
- **Different Ordered Pair:** The sequence in the pair is important and if we write the pair as (e, a) it becomes a diff. ordered pair. Two ordered pair (a, d) and (b, e) are called equal, or the same, if $a = b$ and $d = e$.

Note:

1. $A \times B = \Phi$ if $A = \Phi$ or $B = \Phi$.
2. If A has m elements and B has n elements, then A × B has mn elements.

B × A also has mn element. But the elements of B × A need not be the same as the elements of A × B.

We can also define the Cartesian product of more than two sets in a similar way. Thus, if $A_1, A_2, A_3, \ldots\ldots, A_n$ are n sets, we can define their Cartesian product as $A_1 \times A_2 \times \ldots\ldots \times A_n = \{(a_1, a_2, \ldots\ldots, a_n) \mid a_1 \in A_1, \ldots\ldots, a_n \in A_n\}$.

RELATIONS

By a relation we mean a correlation between two collections of objects. Not surprisingly this takes the form of a set of ordered pairs. A set R is a relation if every element of R is an ordered pair. Of course, this means for every $r \in R$, $r = (x, y)$ and we typically write xRy and say that x stands in relation R to y.

The first relation is the empty one; and a Cartesian product X × Y . Another is the relation in X × X given by $\{(x, x) \mid x \in X\}$ which is the relation of equality. 'Belonging' can be expressed as a relation in X × P(X). Consider the pair (x, A) to be in your relation exactly when $x \in A$.

You may recall the two sets that we defined above for any relation R. These sets are known as the domain