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MATHEMATICAL METHODS IN PHYSICS-II

By:

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Sample Preview of the Solved Sample Question Papers

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QUESTION PAPER

(June – 2018)

(Solved)

MATHEMATICAL METHODS IN PHYSICS - II

Time: 1½ hours] *[Maximum Marks: 25*

Note: *All questions are compulsory. However, internal choices are given. The marks for each question are indicated against it. You may use log tables or non-programmable calculators.*

Q. 1. Answer the following parts:

(a) **Solve the equation**

$$(y+2)\frac{dy}{dx}+x=0 \text{ and}$$

plot your result in *x–y* plane.

Ans. $(y + 2)\, dy/dx + x = 0$

$$dy/dx = -\frac{x}{(y+2)} \qquad -(1)$$

$$\therefore \qquad y = vx$$

$$= dy/dx > u + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = -\frac{x}{(vx+2)}$$

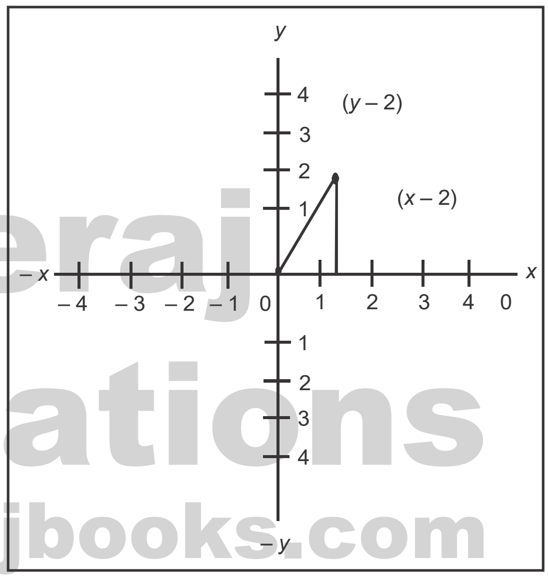
$$x\frac{dv}{dx} = -\frac{x}{(vx+2)} - v$$

$$x\frac{dv}{dx} = \frac{-x-v^2x-2}{vx+2}$$

$$\frac{xdv}{dx} = \frac{(x+v^2x+2)}{(vx+2)}$$

$$\boxed{\frac{dv}{dx} = \frac{(x+v^2x+2}{(vx^2+2)}}$$

Where $\qquad x = 2$ and $y = 2$



(b) **The general solution of the equation**

$$\frac{d^2y}{dx^2}+9y=0$$

is $y(x) = A \sin 3x + B \cos 3x$.

Identify its two solutions and state the condition for their linear independence. Are these solutions linearly independent? Justify your answer by working out steps.

Ans. $d^2y/dx^2 + 9y = 0$

$$y'' + 9y = 0$$

Given $\qquad y_1 = \sin 3x$

and $\qquad y_2 = \cos 3x$

$\therefore \qquad y_1^1 = 3\cos 3x$

and $\qquad y_2^1 = -3\sin 3x$

$\therefore$ their working steps

$$w(x) = \begin{vmatrix} \sin 3x & \cos 3x \\ 3\cos 3x & 3\sin 3x \end{vmatrix}$$
$$= -3\sin^2 3x - 3\cos^2 3x$$
$$= -3\left|\sin^2 3x + \cos^2 3x\right|$$
$$= -3 \times 1$$
$$= -3 \neq 0$$

$\therefore$ y_1, y_2 are linearly independent.

***(c)* Classify the singular points of the equation**

$$x^2 \frac{d^2 y}{dx^2} + x\frac{dy}{dx} + \left(x^2 - \frac{1}{4}\right) y = 0$$

and obtain the corresponding indicial equation.

Ans. $x^2 xy'' + xy' + (x^2 - 1/4)\, y = 0$

$\therefore$ $y'' + 1/x\, y' + (1 - 1/4\, x^2)\, y = 0$

$\therefore$ $p(x) = 1/x + (q/x) = 1 - 1/4\, x^2$

$\therefore$ Singlar point $= 0, \pm$ ½

then $\underset{x \to 0}{\text{LT}}\; xp(x) = \underset{x \to 0}{\text{LT}}\; x\, 1/x = 1$

$x =$

and $\underset{x \to 0}{\text{LT}}\; x^2 q(x) = \underset{x \to 0}{\text{LT}}\; x^2 (1 - 1/4\, x^2) = a -$

$1/4 = 1/4$

$\therefore$ Both $p(x)$, $q(x)$ are analytic at $x = c$, though not at $x = 0$

Let $y(x) = \sum_{n=0}^{\infty} a_m x^{m+r}$

$$\therefore \quad y' = \sum_{m=0}^{\infty} {}^a m\,(m+r)\, x^{m+r-1}$$

$$\therefore \quad y'' = \sum_{m=0}^{\infty} {}^a m\,(m+r)(m+r-1)\, x^{m+r-2}$$

Putting these in given Δ.E. we get

$$= x^2 \Sigma^{a}{}_{m} (\mathrm{M}+r)(m+r-1)\, x^{m+r-2} + x$$

$$\sum_{m=0}^{\infty} a_m (m+r) x^{a_m x^{(m+r)}} + (x^2 - 1/4)$$

$$\sum_{m=0}^{\infty} a_m (m+r) x^{a_m x^{(m+r)}} = 0$$

$$= \Sigma a_m (m+r)(m+r-1+1)\; x^{m+r} + \Sigma a_m x^{m+r+2^{-1/4}} = 0$$

$$= \sum a_m x^{m+r} = 0 = \Sigma a_m \left[(m+r)^2 - 1/4\right]\; x^{m+r} +$$

$$\Sigma a_m x^{m+r+2} = 0$$

from $m = 0$

$$= a_0 [(0+r)^2 - 1/4]\, x^r = 0$$
$$= a_0 (r^2 - 1/4) = 0$$

$r^2 - 1/4 = 0$ is incidial equation.

$\therefore$ $r - \pm$ ½ are roots.

***(d)* The steady state temperature distribution in a control rod in a nuclear reactor is given by:**

$$\frac{\partial^2 \mathbf{T}}{\partial r^2} + \frac{1}{r}\frac{\partial \mathbf{T}}{\partial r} + \frac{\partial^2 \mathbf{T}}{\partial z^2} = \mathbf{0}$$

Use the method of separation of variables to reduce it to a set of ODEs.

Ans. $\dfrac{\partial^2 \mathrm{T}}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial \mathrm{T}}{\partial r} + \dfrac{\partial^2 \mathrm{T}}{\partial z^2} = 0$

Let $\mathrm{T}(r, z) = \mathrm{R}(v).\, z(z$

then $\dfrac{\partial \mathrm{T}}{\partial r} = \dfrac{\partial \mathrm{R}}{\partial r} z$

$$\frac{\partial^2 \mathrm{T}}{\partial r^2} = \frac{\partial^2 \mathrm{R}}{\partial r^2} z$$

$$\frac{\partial^2 \mathrm{T}}{\partial r^2} = \frac{\partial^2 \mathrm{R}}{\partial r^2} z$$

and $\dfrac{\partial \mathrm{T}}{\partial z} = \mathrm{R}\dfrac{d\,\mathrm{T}}{dz}$

and so $\dfrac{\partial \mathrm{T}}{\partial z} = \mathrm{R}\dfrac{d\,\mathrm{T}}{dz}$

Putting in given PPE:

$$z\frac{\partial^2 \mathrm{R}}{dt^2} + \frac{z}{r}\frac{d\mathrm{R}}{dv} + \mathrm{R}\frac{d^2 z}{dz^2} = 0$$

$$\frac{1}{\mathrm{R}}\left[\frac{d^2\mathrm{R}}{dr^2} + \frac{1}{r}\frac{d\mathrm{R}}{dr}\right] = -\frac{1}{z}\frac{d^2 z}{dz^2}$$

As L.H.S. has independent variable r only and R.H.S. has z only so each can be equated to zero.

$$\frac{1}{\mathrm{R}}\left[\frac{d^2\mathrm{R}}{dr^2} + \frac{1}{r}\frac{d\mathrm{R}}{dr}\right] = k \text{ and } -\frac{1}{z}\frac{d^2 z}{dz^2} = k$$

$$\frac{d^2\mathrm{R}}{dr^2} + \frac{1}{r}\frac{d\mathrm{R}}{dr} = k\mathrm{R} \text{ and } \frac{d^2 z}{dz^2} + kz = 0$$

$$\frac{d^2\mathrm{R}}{dr^2} + \frac{1}{r}\frac{d\mathrm{R}}{dr} - k\mathrm{R} = 0 \text{ and } \frac{d^2 z}{dz^2} + 4z = 0$$

which can easily be solved using methods of one variable difference equations.

***(e)* Define the order and degree of a PDE. Write down the orders and degree of the following PDEs.**

Sample Preview
of
The Chapter

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MATHEMATICAL METHODS IN PHYSICS-II

ORDINARY DIFFERENTIAL EQUATIONS

First Order Ordinary Differential Equations

1

INTRODUCTION

Newtons first law defines 'force' using 'rest' and 'motion'. Now mostly Einstein's Theory of Relativity is assumed to have evaporated rest from Universe. I shall however say this— If Universe originates from big bang then Universe is at rest before or at big bang. Hence, absolute rest is not ruled out by Theory of Relativity too. Hence, only change is permanent, is not true.

The Tough times that I treat as a higher theory, however, defines a Universal rest, i.e., zero speed thought which gives Laws of Nature to the Universe. The greatest beauty of Newtonian Mechanics is use of Calculus and the same has continued into Einsteinian. You understand $\frac{dy}{dx}, \frac{d^2y}{dx^2}, \frac{d^3y}{dx^3}$ ...etc., as differential coefficients of 1st, 2nd, 3rd, … etc., order of a function $y = f(x)$ respectively. Hence, any equation involving at least one of them is called a differential equation. Inverse operation of differentiation is integration. Hence, finding back a function from a differential equation by process of integration will give us a general solution of the differential equation which will involve a constant. If a definite value of this constant could be obtained then the solution will be knwon as particular solution. So a differential equation is with one or more of derivatives and variables (one or more of variables or none).

CHAPTER AT A GLANCE

WHAT IS A DIFFERENTIAL EQUATION ?

A differential equation with a single independent variable and one or more dependent variables i.e., $x = f,\left(y\frac{dy}{dx}\right)$ is an ordinary differential equation.

Partial Differential Equation: It involves partial derivatives $\left(\text{e.g.,} \frac{\partial k}{\partial t}, \frac{\partial y}{\partial t} \text{etc.,}\right)$ is called Partial Differential Equation.

CLASSIFICATION OF ORDINARY DIFFERENTIAL EQUATION

Order of an ODE is order of highest derivative appearing in it.

The degree is the highest power of the highest order derivative in the ODE after expressing it in a form such that no derivatives have fractional or negative powers.

Examples:

(*i*) Degree and order of $L\frac{di}{dt} + Ri = E$ are |and| respectively. It is a linear and non-homogeneous equation, however can be made homogeneous as follows:

$$L\frac{di}{dt} = E - Ri \Rightarrow \frac{di}{dt} = \frac{E - Ri}{L}$$

Now putting $i_1 = i - \frac{E}{R}, = \frac{di_1}{dt} = \frac{di}{dt}$

$\therefore$ The equation becomes

$$\frac{di_1}{dt} = \frac{E-(Ri_1+E)}{L} = \frac{-Ri}{L} \Rightarrow \frac{di_1}{dt} = \frac{-Ri}{L}$$

$$\Rightarrow \quad \frac{di_1}{i_1} = \frac{-R}{L}dt$$

Integrating it we get

$$\log i_1 = \frac{-R}{L}t + K$$

(*ii*) $m\frac{d^2x}{dt^2} + r\frac{dx}{dt} + Kx = F\cos\omega t$ is of second order and degree 1.

(*iii*) $x^3 y'' + 3xy' = y$ is of order 2 and degree 1. It is homogeneous, linear ODE.

(*iv*) $y'' = [1 + (y)^2]^{3/2} \Rightarrow (y'')^2 = [1 + (y')^2]^3$

$\therefore$ Now differential coefficient has negative or fractional degree any more and so we find the order to be 2 and also degree as 2.

(*v*) $\frac{d^3y}{dx^3} - y = e^x$ is non-homogeneous, of degree 1 and order 3.

(*vi*) $\frac{d^2y}{dx^2} - 9y = \cos y$ is non-homogeneous of order 2 and degree 1.

(*vii*) $(y'')^4 + 2xy' - y = 0$ is of order 2 and degree 4.

(*viii*) $y''' - 3y'' + 4y = 0$ is of order 3 and degree 1.

WHAT IS A SOLUTION OF A DIFFERENTIAL EQUATION ?

Suppose we take

$$y^2 + 2x = 8$$

then $\quad 2yy' + 2 = 0 \Rightarrow yy' = -1.$

Here $\quad y = \pm\sqrt{8-2x}$ gives $yy' = -1$ and hence $y = \pm\sqrt{8-2x}$ are two solutions of $yy' = -1$.

Here $x \in [-4, 4]$

Similarly

$y'' + y = 0$ has a solution

$y = \cos x$ as

$$y' = -\sin x \text{ and } y'' = -\cos x$$

$$\Rightarrow y'' = -y \Rightarrow y'' + y = 0$$

$\therefore \quad y'' + y = 0$ has a solution $y = \cos x$

Obviously the domain in this case is $(-\infty, \infty)$

Hence, ac function $y = f(x)$ is a solution of a differential equation in y on some interval, say $a \le x \le b$, if $f(x)$ is defined and differentiable throughout that interval and is such that the equation becomes an identity when y is replaced by $\phi\, f(x)$ in the DE.

A solution of form $y = f(x)$ is an explicit form of solution whereas $F(x, y) = 0$ is of an implicit form.

General Solution and Particular Solution

Let us consider $\quad y' = -\sin x_i$ we note

$y = \cos x$, $y = \cos x + 5$,

$y = \cos x - \frac{8}{9}$ etc., all satisfy

$y' = -\sin x$

They are all solutions of $y' = -\sin x$. They can all be written as $y = \cos x + c$ where c = an arbitrary constant. Such a solution is called a general solution.

Similarly $y'' + y = 0$ has

$y = A\cos x + B\sin x$ as general solution.

General solution is the solution of a D.E. involving arbitrary constant(s).

The number of arbitrary constants is equal to the order of D.E.

Particular Solution: Suppose we impose $y = 0$ at $x = 0$ on $y' = -\sin x$ then in $y = \cos x + c$, putting $x = 0$, $y = 0$, gives

$$0 = \cos 0 + c \Rightarrow c = -1$$

$\therefore$ $y = \cos x - 1$ is the particular solution under the conditions imposed and hence, if a definite value can be assigned to each arbitrary constant appearing in a general solution, then we get a particular solution.

Initial Value Problems: If the conditions on the solution of a DE or its derivatives are specified for a single value of independent variable, they are called initial conditions. A DE with initial conditions is called an initial value problem.

Boundary Value Problems: If the conditions on the solution of DE or its derivatives are specified for more values of independent variable, they are called boundary conditions. The DE with its boundary condition is called boundary-value problem.

Existence and Uniqueness of a Particular Solution: *n*th order ODE contains *n* arbitrary constants, hence we have to impose n conditions on the solution functions and its derivatives. For this, there are two common methods:

1. If the conditions on the solution of an ODE or its derivatives are specified for a single value of the independent variable and these conditions and the ODE together are called an Initial Value Problem (IVP).

2. If the conditions on the solution of an ODE or its derivatives are specified by two or more values of the independent variable, they are called a boundary value problem alongwith ODE. e.g.

(*i*) $y' + 3y = 4$ with initial condition $y(0) = 1$ is first order initial value problem.

(*ii*) $y'' + 4y = 0$ with initial condition $y(1) = 2$, $y'(1) = 4$ is a second order initial-value problem.

(*iii*) $y'' - 4y' + 4y = x^2$ with $y(0) = 2$ and $y'(1) = -1$ is a second order boundary value problem.

Examples:

(*i*) $y' + 2y = 3$ with $y(0) = 1$ is a first order initial-value problem.

(*ii*) $y'' + 3y = 0$ with the initial conditions $y(1) = 2$ and $y'(1) = -8$ is a second-order initial-value problem.

(*iii*) $y'' - 2y' + 6y = x^3$ with the boundary conditions $y(0) = 2$, $y(1) = -1$ is a second-order boundary-value problem.

(*iv*) $y = A\cos x + B\sin x$ is the general solution of $y'' + y = 0$, let $y(0) = 0$, $y(\pi) = 2$ then integrating

$$y'' + y = 0 \text{ to get}$$
$$y = A\cos x + B\sin x \text{ and}$$
$$y' = -A\sin x + B\cos x$$

Putting $y(0) = 0$, $y(\pi) = 2$ in above, we get

$0 = A\cos 0$ and $2 = A\cos\pi + B\sin\pi$

$\therefore$ $A = 0$ and $2 = -A + 0 \Rightarrow A = -2$

So we get contradictory values of A which is not possible

$\therefore$ Above boundary-value problem can not be solved. So it is not possible to solve every differential equation.

(*v*) Let us consider conditions $y(0) = y(\pi) = 0$ with DE $y'' + y = 0$. $y = A\cos x + B\sin x$ is its general solution. We put $y(0) = 0$ and $y(\pi) = 0$ in it.

$0 = A\cos 0 + B\sin 0$ and $0 = A\cos\pi + B\sin\pi$

$\therefore$ $A = 0$ and $0 = -A + B.0 \Rightarrow A = 0$

So only one arbitrary constant A is obtained and not B. Hence boundary line problem has a solution.

$y = B\sin x$ and represents infinitely many solutions for values of B.

Unique Solution: It is not possible to solve all boundary value problems as seen above. Moreover, if it exists, it may not be unique.

General Properties of Solution of Linear ODEs

Two second order ordinary differential equations

$$a_2(x)y'' + a_1(x)y' + a_0(x)y = 0 \quad \ldots(1)$$

and

$$a_2(x)y'' + a_1(x)y' + a_0(x)y = f(x) \quad \ldots(2)$$

are both linear but first (1) is homogeneous and second (2) is non-homogeneous.

Properties of the solutions of Linear ODEs:

1. $y = 0$ is solution of (1) and it is called trivial solution.

2. If y_1 and y_2 are linearly independent solutions of (1) then $y = c_1y_1 + c_2y_2$ is also its solution, c_1, c_2 being constants.

3. If y_1 is a solution of (1) and y_2 is a solution of (2). Then $z = y_1 + y_2$ is a solution of (2).

4. $z = y_1 - y_2$ is a solution of (2) if y_1, y_2 are two solutions of (2).

EQUATIONS REDUCIBLE TO SEPARABLE FORM

Method of Separation of Variables

Let us take a general first order ODE

$y' = f(x, y)$ which can be written as

$$f(x, y) = \frac{M(x)}{N(y)} = \frac{dy}{dx}$$

then $M(x)\,dx - N(y)\,dy = 0$

This can be integrated as follows

$$\int M(x)\,dx - \int N(y)\,dy = C$$

Example. Solve the equation $\dfrac{dy}{dx} = \dfrac{8(x^2+2)}{xy}$.

Sol. We write it

$$y\,dy = \frac{8(x^2+2)}{x}\,dx$$

i.e.,

$$y\,dy = 8\left(x + \frac{2}{x}\right)dx$$

Integrating it

$$\frac{y^2}{2} = 8\left[\frac{x^2}{2} + 2\ln x\right] + \frac{C}{2}$$

$$\Rightarrow y^2 = 8x^2 + 32\ln x + C$$

Solution by the Method of Substitution

Substitution by inspection e.g. in following:

$$\frac{dy}{dx} = \sin(x+y),$$

we can put $z = x + y$ and solve as follows:

$$z = x + y \Rightarrow \frac{dz}{dx} = 1 + \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{dx}{dx} - 1$$

$$\therefore \quad \frac{dz}{dx} - 1 = \sin z \Rightarrow \frac{dz}{dx} 1 + \sin z$$

which can be solved by variable separable method as

$$\frac{dz}{1+\sin z} = dx \Rightarrow \frac{dz}{1+\cos\left(\frac{\pi}{2} - z\right)} = dx$$

$$\frac{dz}{1+2\cos^2\left(\frac{\pi}{4} - \frac{z}{2}\right) - 1} = dx \text{ and so integrating}$$

$$\frac{1}{2}\int \sec^2\left(\frac{\pi}{4} - \frac{z}{2}\right) - dz = \int dx + \text{C}$$

$$\frac{\frac{1}{2}.\tan\left(\frac{\pi}{4} - \frac{z}{2}\right)}{-\frac{1}{2}} = x + \text{C}$$

$$-\tan\left(\frac{\pi}{4} - \frac{z}{2}\right) = x + \text{C}$$

$$\Rightarrow x + \tan\left[\frac{\pi}{4} - \left(\frac{x+y}{2}\right)\right] + \text{C} = 0$$

which is required general solution.

Homogeneous Differential Equations of the First Order

What is a homogeneous function?

Let $z = f(x, y)$

If $f(\lambda x, \lambda y) = \lambda^n f(x, y)$
$= \lambda^n z$

then $z = f(x, y)$ is a homogeneous function of degree n, n can be any real number.

(a) $f(x, y) = x^2 + 3xy + y^2$

$\therefore \quad f(\lambda x, \lambda y) = \lambda^2 x^2 + 3\lambda^2 xy + \lambda^2 y^2$
$= \lambda^2(x^2 + 3xy + y^2)$
$= \lambda^2 f(x, y)$

$\therefore f(x, y)$ is a homogeneous function of degree 2.

(b) $f(x, y) = \sqrt{x + 3y}$

$\therefore \quad f(\lambda x, \lambda y) = \sqrt{\lambda x + 3\lambda y}$
$= \sqrt{\lambda}\sqrt{x + 3y}$
$= \lambda^{1/2} f(x, y)$

$\therefore f$ is homogeneous function of degree $\frac{1}{2}$.

(c) $f(x, y) = \sin \frac{x}{y} + e^{y/x}$

$$\therefore \quad f(\lambda x, \lambda y) = \sin\left(\frac{\lambda x}{\lambda y}\right) + e^{\frac{\lambda y}{\lambda x}}$$

$$= \lambda^{\circ}\left[\sin\frac{x}{y} + e^{y/x}\right]$$

$$= \lambda^{\circ} f(x, y)$$

$\therefore f$ is homogeneous of degree.

(d) $f(x, y) = \cos\left(\sqrt{\frac{x+3y}{y}}\right) + \frac{x}{y}$

$$\therefore \quad f(\lambda x, \lambda y) = \cos\left(\sqrt{\frac{\lambda x + 3\lambda y}{\lambda y}} + \frac{(\lambda x)}{y\lambda}\right)$$

$$= \cos\sqrt{\frac{x+3y}{y}} + \frac{x}{y} = f(x, y)$$

$\therefore f$ is homogeneous of degree 0.

Hence to solve a homogeneous first order DE, we put $y = x$.

However if $\frac{dy}{dx} = \frac{a_1 x + b_1 y + c_1}{a_2 x + b_2 y + c_2}$ then we put

$x = \text{X} + h$ and $y = \text{Y} + k$ and then

$$\frac{d\text{Y}}{d\text{X}} = \frac{dy}{dx} = \frac{a_1\text{X} + b_1\text{Y} + (a_1 h + b_1 k + c_1)}{a_2\text{X} + b_2\text{Y} + (a_2 h + b_2 k + c_2)}$$

Here by putting $a_1 h + b_1 k + c_1 = 0$ and $a_2 h + b_2 k + c_2 = 0$, and solving them, we find h, k

$$\therefore \frac{dy}{dx} = \frac{a_1\text{X} + b_1\text{Y}}{a_2\text{X} + b_2\text{Y}} \text{ is homogeneous DE}$$

It can be solved as before and then by replacing $\text{X} = x - h$, $\text{Y} = y - k$, we get the solution.