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QUESTION PAPER

June – 2024

(Solved)

QUANTITATIVE METHODS

MEC-203

Time: 3 Hours] *[Maximum Marks: 100*

Note: *Answer questions from each Section as per instructions.*

SECTION-A

Note: **Answer any two questions from this Section.**

Q. 1. ***(a)*** **Use Cramer's rule to find the solution of the following system of equations:**

$$-X_1 + 4X_2\ 3X_3 = 2$$
$$2X_2 + 2X_3 = 1$$
$$X_1 - 3X_2 + 5X_3 = 0$$

Ans. To solve the system of equations using Cramer's rule, follow these steps. The system of equations is:

(b) **Discuss the properties of orthogonal matrix and idempotent matrix.**

Ans. Ref.: See Chapter-5, Page No. 46, Q.No. 9 and Q. No. 10.

(c) **Define the terms eigen value, eigen vector and characteristic equation.**

Ans. Ref.: See Chapter-7, Page No. 56, Q.No. 1.

Q. 2. ***(a)*** **Explain Taylor's approach to polynomial approximation.**

Ans. Ref.: See Chapter-14, Page No. 114, 'Taylor's Approach to Polynomial Approximation'.

(b) **Let $F = R^3 \to R$ such that $\bar{x} = (x_1, x_2, x_3)$. Let $F(\bar{x}) = e^{x_1 + x_2 + x_3}$. Find Taylor's third order polynomial in the neighbourhood of (0, 0, 0).**

Ans. The system given is:

$$-x_1 + 4x_2 + 3x_3 = 2,$$
$$0x_1 + 2x_2 + 2x_3 = 1,$$
$$x_1 - 3x_2 + 5x_3 = 0.$$

This can be written in matrix form as:

$$A = \begin{bmatrix} -1 & 4 & 3 \\ 0 & 2 & 2 \\ 1 & -3 & 5 \end{bmatrix}, X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, B = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}.$$

According to Cramer's rule, the solution is given by:

$$x_i = \frac{\det(A_i)}{\det(A)}$$

where A_i is the matrix A with the *i*-th column replaced by the column matrix B.

Step 1: Compute det (A)

$$\det(A) = \det \begin{bmatrix} -1 & 4 & 3 \\ 0 & 2 & 2 \\ 1 & -3 & 5 \end{bmatrix}$$

Step 2: Compute det (A_1), det (A_2), and det (A_3)

Matrix A_1 (replace first column by B):

$$A_1 = \begin{bmatrix} 2 & 4 & 3 \\ 1 & 2 & 2 \\ 0 & -3 & 5 \end{bmatrix}$$

Matrix A_2 (replace second column by B):

$$A_2 = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 2 \\ 1 & 0 & 5 \end{bmatrix}$$

Matrix A_3 (replace third column by B):

$$A_3 = \begin{bmatrix} -1 & 4 & 2 \\ 0 & 2 & 1 \\ 1 & -3 & 0 \end{bmatrix}$$

Step 3: Compute determinants and find x_1, x_2, and x_3

Let's calculate det (A), det (A_1), det (A_2), and det (A_3) to find the variables x_2, x_2, and x_3.

The determinants calculated are as follows:

- det (A) = – 14
- det (A_1) = 3
- det (A_2) = – 4
- det (A_3) = – 3

Using Cramer's rule, we can find x_1, x_2, and x_3 using the formula $x_i = \frac{\det(A_i)}{\det(A)}$:

$$x_1 = \frac{\det(A_1)}{\det(A)} = \frac{3}{-14} = -\frac{3}{14}$$

$$x_2 = \frac{\det(A_2)}{\det(A)} = \frac{-4}{-14} = \frac{2}{7}$$

$$x_3 = \frac{\det(A_3)}{\det(A)} = \frac{-3}{-14} = \frac{3}{14}.$$

Q. 3. ***(a)*** **What do you understand by sample design? List the advantages of sample survey.**

Ans. Ref.: See Chapter-28, Page No. 206, 'Sample Design and Advantages of Sample Survey'.

(b) **Briefly explain different types of sampling methods.**

Ans. Ref.: See Chapter-28, Page No. 207, 'Types of Sampling'.

(c) **What are the different sources of bias in sample survey?**

Ans. Ref.: See Chapter-28, Page No. 207, 'Biases in the Survey'.

Q. 4. Find the time path and investigate the behaviour of price in a market when demand and supply functions are:

$D_t = 86 - 0.8\,P_t$

$S_t = -10 + 0.8\,P_{t-1}$

Ans. We are given demand and supply functions in the market and are tasked with finding the time path of price, as well as investigating the behavior of price over time.

The demand and supply functions are as follows:

– Demand: ($D_t = 86 - 0.8P_t$)

– Supply: ($S_t = -10 + 0.8P_t$)

Step 1: Determine the equilibrium price: At equilibrium, the quantity demanded equals the quantity supplied:

$$D_t = S_t$$

Substituting the given demand and supply functions:

$$86 - 0.8P_t = -10 + 0.8P_t$$

Step 2: Solve for the equilibrium price: Simplifying the equation:

$$86 + 10 = 0.8P_t + 0.8P_t$$

$$96 = 1.6P_t$$

$$P_t = \frac{96}{1.6} = 60$$

So, the equilibrium price is ($P_t = 60$).

Step 3: Time path of price: To explore the time path and behavior of price over time, we assume a dynamic adjustment process in the market. A common approach is to assume that the price adjusts over time according to the difference between demand and supply.

Let's assume a simple price adjustment rule as follows:

$$\frac{dP_t}{dt} = \alpha\,(D_t - S_t)$$

Where:

– (α) is a positive constant representing the speed of price adjustment.

– ($D_t - S_t$) is the excess demand.

Substitute the demand and supply functions into the adjustment equation:

$$\frac{dP_t}{dt} = \alpha\,((86 - 0.8P_t) - (-10 + 0.8P_t))$$

Simplify:

$$\frac{dP_t}{dt} = \alpha\,(96 - 1.6\,P_t)$$

This is a first-order linear differential equation.

Step 4: Solve the differential equation: To solve this equation, separate the variables:

$$\frac{dP_t}{96 - 1.6\,P_t} = \alpha dt$$

Now, integrate both sides:

$$\int \frac{dP_t}{96 - 1.6\,Pt} = \alpha \int dt$$

The left-hand side is a standard logarithmic integral:

$$-\frac{1}{1.6}\ \ln|96 - 1.6P_t|\ \alpha t + C$$

Where (C) is the constant of integration. Multiply both sides by (– 1.6) to simplify:

$$\ln|96 - 1.6P_t| = -1.6\alpha t + C'$$

Sample Preview of The Chapter

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QUANTITATIVE METHODS

Fundamental Concepts of Mathematics 1

INTRODUCTION

The focus of the discussion in this chapter is on the two fundamental ideas in mathematics: *(i)* Number, which forms the basis of traditional mathematics, and *(ii)* Set, which is the base of contemporary mathematics.

Let's first comprehend the idea, phrase, name, symbol, and method of the quantity.

Fig.: 1(a) Fig.: 1(b) Fig.: 1(c)

Number Concept: If one sees Figures 1*(a)* and 1*(b)* as being distinct from each other and Figures 1*(b)* and 1*(c)* as being comparable to each other, then one has number-sense, or the notion or concept of numbers (of at least up to 2). The number sense/concept reveals that despite the similarities between the two figures in Figures 1*(a)* and 1*(b)*, such as the fact that both are about horses and even black horses, they are distinct in one important way. In a similar way, although having many differences – Fig. 1*(b)* is a photo of real black horses, whereas, Fig. 1*(c)* is a picture of marble white elephants (toys) – the two images have the quality of "twoness." 'Twoness' is a notion.

Number name/Term: For the number of horses in Fig. 1*(b)*, there are different names/terms in different languages: 'two' in English, 'दो' in Hindi, 'zwei' in German, 'இரண்டு' or *'Iraṇṭu'* in Tamil.

Concept & Term: A concept's name is referred to as a "term."

For example, the terms of "twoness" are represented by the phrases "two" and "."For this **concept,** alternative words like "pair," "couple" and even "twin" may occasionally be employed, even in English. The term "bank," on the other hand, can be used to describe to two distinct ideas, namely, a river and "the place where money is deposited," among other things.

Numeral/Number-Notation/Number-Symbol: There are several well-known notations for the number "six," including "6," "VI," and "." Thus, 6 is a numeral and not a real number; it is the name of a number. However, it is misnomered as a number. It is customary in mathematics to refer to numerals as numbers. In place of the actual numerical, we will also typically use the phrase "number."

Term, Notation and Definition: One of the mathematical expressions is "square root," and the notation or symbol for it is √. In addition, the definition of square-root reads, "For a given number x, its square-root y is a number that satisfies the equation y x $y = x$, where the meaning of "x" or multiplication is already known."

Numeral System (or System of Numeration): Numerous number systems – or, more properly, numeral systems – have been mentioned before, including the decimal, binary, and Roman numeral systems. The decimal number 41, for instance, can be written as 'XLI' in the Roman numeral system and '101001' in the binary number system. The same statement may be used to represent several numbers depending on the number system. As an illustration, the symbol 111 stands for "One Hundred and Eleven" in the decimal number system, "Seven" in the binary number system, and "Three" in the uniary number system.

CHAPTER AT A GLANCE

SET THEORY

Traditionally, the most basic and first topics covered in introductory mathematics were the concept of numbers and geometric concepts like "straight line", "triangle", and "square".

Concept of Set

A set can be thought of as a grouping of items or entities, such as the names of animals, books, numbers, geometric shapes, or any combination of these. A set's member or element is any object that is a part of the set.

Notations

Which is indicated by the symbol '{'comes first in the set notation, followed by one of the elements, such as 0, a comma, and a blank space. This pattern continues until the last element is written, which is then followed by the right brace, which is indicated by the symbol "}". Typically, a set is given a name.

Forms of Set Representation

Two popular ways to represent sets are as follows:

***(i)* Tabular or Roster Form:** This notation has been used up to this point to indicate a set. Each element or member in this form must be enclosed in braces, or there must be an adequate number of members to show a pattern.

***(ii)* Set-builder Form:** The set–builder form uses these properties to denote the set as follows:

Sq_int_less_10001= $\{x: x = y^2$, y is an integer, and $x < 10001\}$

Relationships between Sets

Here, we show how to build each of these set relationships using a straightforward example. Later on, we'll look at some examples of building these relationships that are more complicated, especially with regard to the equality (=).

Notation, Definition and Examples

(i) If every element in both sets, X and Y, is also an element in X, then the two sets are said to be equal.

For "equality of sets X and Y," utilise the notation X= Y.

(ii) If each element in X is also an element in Y, then X is a subset of Y. We can also say "Y is a superset of X," "X is contained in Y," or "Y contains X" in place of "X is a subset of Y."

(iii) If *(i)* each element in X is an element of Y and *(ii)* there is at least one element in Y that does not belong to X, then X is a proper subset of Y. Then, Y is a proper superset of X, which is likewise true.

Establishing Set Relationship

Let's think about sets.

A = $\{x: x^2+3x + 2 = 0\}$, the set of solutions of the equation $x^2+ 3x + 2 = 0$.

B = { −1, −2}, C= { −2}, E = {0, −1, −2}

Notation, Definition and Examples

***(i)* Easiest case is that of 'is not a subset of':** In our case, E $\not\subset$ C, as 0 $\in$ E, but 0 $\notin$ C. Similarly, E $\not\subset$ A, as 0 $\in$ E, but 0 is not a root of $x^2 + 3x + 2 = 0$.

(ii) After that, we must B $\subseteq$ E. Evaluate each component of B individually and demonstrate that it is a part of both B and E. In complicated circumstances, we choose one element at random rather than thinking about each element separately. Later on, the methods will be described.

(iii) In addition to what we have already accomplished in *(ii)* above, we also need to demonstrate the existence of an element of E that does not belong to B in order to demonstrate that B is a proper subset of E B $\subseteq$ E. In this instance, 0 $\in$ E, but 0 $\notin$ B.

(iv) In order to demonstrate the equality (=) of two sets, case *(ii)* must be repeated twice with the roles of the sets reversed.

In our case, A= B, because −1, which belongs to B, is a root of $x^2 + 3x + 2 = 0$, hence, belongs to A. Similarly, −2, which belongs to B, is a root of $x^2 + 3x + 2 = 0$, hence, belongs to A. Conversely, as $x^2 + 3x + 2 = 0$ is a quadratic equation, it has exactly two roots, viz. −1, −2.

Some Special Sets: Universal Set, Null Set, Power Set and Convex Set

***(a)* Universal Set:** The Universal set is one such collection and is often U. Consequently, by definition, every set x under consideration will be a subset of the Universal set U.

***(b)* The Empty or Null Set:** The number of elements in a set may be zero. According to our definition of a set, the following is a set S = $\{x: x$ is an even integer, and $x^2= 9\}$.

A null set is denoted by the standard notation Ø or by { }.

Quantifiers and Logical Symbols/Operators

Quantifiers and Logical operators appear frequently throughout mathematics.

***(a)* Quantifiers**

($\forall$x) $x \in x$ denotes 'for all $x \in x$';

($\exists$x) $x \in x$ denotes 'for some $x \in x$';

($\nexists x \in x$) 'there does not exist an x in x' OR 'there is no x in x, which satisfies'.

***(b)* Logical Operators**

A $\rightarrow$ B denotes 'If A then B',

'A$\leftrightarrow$B denotes 'A if and only if B',

'A $\leftrightarrow$B' is equivalent to both A $\rightarrow$B and B $\rightarrow$ A combined.

Some Conceptual Clarifications and Explanations

The various "Relation" notions are contrasted in this subsection & "Operation", Sets operations: union, intersection, difference, and complementation, further clarifications: Operation, operands, argument, inputs, value, result, and output contra operand.

Difference between the Concepts of 'Relation' and 'Operation'

Relation on a set *x* is a subset of cross-product of the *x* with itself *n* times, where n is the arity of the where n is the number of relations in the set. For instance, the relation is-male involves a single person, making it a relation of arity 1, whereas the relation *is-mother-of* involves two people, making it a relation of arity 2. A relation returns exactly one of "true" or "false" as a value.

Operation on a set *x* is quite similar to that of relation. The arity of the operation is determined by the number of times an operation on *x* maps the cross-product of *x* with itself. However, it gives back one element of *x* itself as value.

Operations on Sets: Union, Intersection, Difference, Complementation

(a) **Union $x \cup y$ of two sets *x* and *y* is defined** as:

$x \cup y = \{x: x \in x$ or $x \in y\}$ Here, $x \in x$ or $x \in y$ also includes the case that *x* may belong to both of *x* and *y*.

For example, if $x = \{1, 2, 3, 4\}$ and $y = \{2, 3, 5, 8, 9\}$, then $x \cup y = \{1, 2, 3, 4, 5, 8, 9\}$

Elements, like 2 and 3, which are occurring in both *x* and *y* appear only once in $x \cup y$, as elements are not repeated in representation of a set.

(b) **The Intersection $x \cap y$ of two sets *x* and *y* is defined as:**

$x \cap y = \{x: x \in x$ and $x \in y\}$

For example, if $x = \{1, 2, 3, 4\}$ and $y = \{2, 5, 8, 3, 9\}$, then$X \cap y = \{2, 3\}$,

(c) **The Difference *x* – *y* of two sets *x* and *y* is defined as:**

$$x - y = \{x: x \in x \text{ and } x \notin y\}$$

For example, if $x = \{1, 2, 3, 4\}$ and $y = \{2, 5, 8, 3, 9\}$, then $x - y = \{1, 4\}$ and $y - x = \{5, 8, 9\}$

(d) **Unary Set operator 'Complement': *x*' denotes 'complement of *x*':** 'Complement of a set *x*' is a specific instance of 'Difference of Two Sets'. We need to know the value of the Universal set U, which is the largest set being taken into account in the specific context, in order to define "complement of a set". If *x* is represented by *x*' and U stands for the Universal set under study, then *x*''s complement is defined as U-X.

More Clarifications

(i) **Operands/Argument/Inputs; Value/result/output:** In one of the examples above, we mentioned if $x = \{1, 2, 3, 4\}$ and $y = \{2, 3, 5, 8, 9\}$, then $x \cup y = \{1, 2, 3, 4, 5, 8, 9\}$.

The values {1, 2, 3, 4} and {2, 3, 5, 8, 9}, which are given as inputs to the operation of union are generally called operands.

(ii) **Operation *vs.* Operator:** The two concepts of 'operation' and 'operator' are quite similar and are often confused for each other. For our purpose, the symbol for union, namely ∪, is called the operator.

NUMBERS: FROM NATURAL TO COMPLEX

Some Standard Number Sets

N = {1, 2, 3…} is called the set of natural/counting numbers.

W= {0. 1, 2, 3 …} is called the set of whole numbers

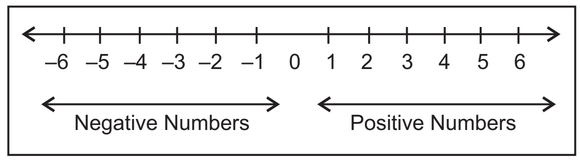
Z = {…, − 3, − 2, − 1, 0, 1, 2, 3…} is called the set of integers.

$Q = \{x: x = p/q$ with $q \neq 0$, and $p, q \in Z\}$ is called the set of rational numbers.

R, The Set of Real Numbers

The geometric meaning of the set of real numbers symbolized by Rand is described ahead:

Mark two separate locations on a straight line, designating the left mark as 0, and the right mark as 1. Then, it is possible to have each point on the straight line (stretched infinitely in both directions) correspond to a real integer.



Real Number Intervals

The set of all real numbers between 3 and 7.5 is designated by the interval [3, 7.5], and it also includes both the boundary numbers 3 and 7.5. A closed interval is one such interval.

The set of all real numbers within the interval [3, 7.5] (sometimes written as [3, 7.5]) is called numbers from 3 to 7.5, including 7.5, but not 3.

Complex Numbers

The set of complex numbers developed in context of attempts at solving equations of the type $x^2 + 1 = 0$, for which no real number solution exists. A new type of number i was introduced for which $i^2 = -1$ was assumed. Then C is defined as $C = \{z: z = x + i\, y: x, y \in R\}$.

Number Miscellany

Number (**Examples:** 3, −12, 8.7, 2/3, √2, π, and 7 – 3*i*, where 3 is a natural number, − 12 is an integer, each of 8.7 and 2/3 is a rational number, each of √2 and π is an irrational number. Each of the numbers 3, −12, 8.7, 2/3, √2, π.

Properties of Real Numbers as Ordered Field

(a) R, the set of real numbers, satisfies the following properties (as a Field) w.r.t the two binary operations '+' and '×':

1. Closure Properties: $x + y \in R$, and $x \times y \in R$,

2. Associative Properties: $(x + y) + z = x + (y + z)$; $(x \times y) \times z = x \times (y \times z)$

(b) R, the set of real numbers, additionally satisfies the following properties (as an Ordered Field) w.r.t the two binary operations '+' and '×', and the binary relation '<':

DECIMAL REPRESENTATION OF RATIONAL AND IRRATIONAL NUMBERS

(i) **The terminating decimal representation of rational:** Think about 3/8. We are aware that it is written as 0.375 in decimal notation. Due to the fact that it only has a finite amount of digits, this form is terminating.

(ii) **Non-terminating recurring/repeating decimal representation of rational numbers:** 7/3 is written in decimal notation as 2.333..., where the digit 3 repeats itself indefinitely. This means that the decimal representation is non-terminating, yet the sequence only consists of one repeating digit, 3.

REAL NUMBER EXPONENTIATION

We know that the notation x^3 stands for $x. x. x$, i.e. x multiplied by itself three times. In general, for $n \in N$, the notation x^n stands for $x. x\ldots$ (n times), i. e., x multiplied by itself n times. Then x is called the base of x^n and n is called the exponent or power of x^n.

CHECK YOUR PROGRESS

Q. 1. Which of the following collections are sets? Justify your answer.

(i) **The collection of all the months of a year beginning with the letter M.**

(ii) **The collection of all the months of a year beginning with the letter Z.**

(iii) **The collection of ten most talented living writers of the world.**

(iv) **A team of eleven most renowned football players of the world.**

(v) **{1, 2, 2, 3}**

(vi) **{1, 2, {2, 3}, 3}**

Ans. *(i)* The collection of all the months of a year beginning with the letter M.

This is a set because it contains distinct elements: {March, May}

(ii) The collection of all the months of a year beginning with the letter Z.

This is an empty set because there are no months that start with the letter Z. An empty set is still a set.

(iii) The collection of ten most talented living writers of the world.

Whether this is a set, or not depends on whether there is a specific, agreed-upon list of the ten most talented living writers. If such a list exists and contains distinct writers, then it can be considered a set. If not, it may not be a set.

(iv) A team of eleven most renowned football players of the world.

Similar to *(iii)*, whether this is a set or not depends on whether there is a specific, agreed-upon list of the eleven most renowned football players. If such a list exists and contains distinct players, then it can be considered a set. If not, it may not be a set.

(v) {1, 2, 2, 3}

This is not a set, because it contains duplicate elements (the number 2 appears twice). Sets cannot contain duplicate elements.

(vi) {1, 2, {2, 3}, 3}

This is not a set, because it contains a nested set ({2, 3}). Sets typically consist of individual, non-nested elements. If you remove the nested set, you have {1, 2, 3}, which is a valid set.

Q. 2. For each of the sets consisting of following members, write it in roster/tabular form and its cardinality:

(a) **Positive prime integer factors of 180**

(b) **Solutions of the quadratic equation $x^2 - 3x - 10 = 0$**

Ans. *(a)* Positive prime integer factors of 180: Prime factors of 180 are 2, 3, and 5. We are looking for positive prime integer factors, so the set can be represented as {2, 3, 5}

The cardinality of this set is the number of elements in it, which is 3. So, the cardinality is 3.

(b) Solutions of the quadratic equation $x^2 - 3x - 10 = 0$:

To find the solutions, we can solve the equation:

$$x^2 - 3x - 10 = 0$$

Factoring the equation:

$$(x - 5)(x + 2) = 0$$

Setting each factor equal to zero:

$$x - 5 = 0 \Rightarrow x = 5, x + 2 = 0 \Rightarrow x = -2$$

The solutions to the equation are $x = 5$ and $x = -2$.

Representing the set of solutions in roster form: {5, –2}

The cardinality of this set is 2, as there are two elements in it. So, the cardinality is 2.

Q. 3. Tell which of the following is a set, and if it is a set, then:

(i) **Write it in set builder form**

(ii) **Tell its cardinality.**

(a) **{6, 9, 12, 15, 18, 21, 24, 27}**

(b) **{2, 4, 8, 16, …}**

(c) **{{1}, {1, 2}, {1, 2, 3}, …, {1, 2, 3, 4…10}}**

Ans. *(a)* {6, 9, 12, 15, 18, 21, 24, 27}

This is a set, because it contains distinct elements. It represents a set of positive multiples of 3, starting from 6.

(i) Set builder form: {x | x is a positive multiple of 3}

(ii) **Cardinality:** There are 8 elements in this set, so the cardinality is 8.

(b) {2, 4, 8, 16, ...}

This is a set. It represents a geometric sequence where each element is a power of 2.

(i) Set builder form: {2^n | n is a non-negative integer}

(ii) **Cardinality:** This set contains infinitely many elements because it includes all powers of 2, so the cardinality is infinite (∞).

c) {{1}, {1, 2}, {1, 2, 3}, …, {1, 2, 3, 4…10}}

This appears to be a set of sets where each set contains the numbers from 1 to some positive integer.

(i) Set builder form: {S | S is a set containing the numbers from 1 to n, where n is a positive integer}.

(ii) **Cardinality:** This set contains infinitely many sets because it includes sets containing numbers from 1 to various positive integers, so the cardinality is infinite (∞).

Q. 4. For the pair of sets x and y, state whether $x = y$ or not, along with explanation for your response.