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Sample Preview of the Solved Sample Question Papers

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QUESTION PAPER

June – 2023

(Solved)

BUSINESS MATHEMATICS AND STATISTICS

B.C.O.C.-134

Time: 3 Hours] ***[Maximum Marks: 100***

Note: *Question* ***No. 1*** *is compulsory. (ii) Answer both Part A and Part B. (iii) All questions carry* ***equal*** *marks.*

Q. 1. ***(a)*** **Discuss application and use of Matrices for Business and Economic decision-making.**

Ans. Ref.: See Chapter-4, Page No. 32, Q. No. 1 and Page No. 34, Q. No. 10 and Q. No. 13.

(b) **Define statistics. Explain the importance and scope of statistics in Business and Economic decision-making.**

Ans. Ref.: See Chapter-12, Page No. 110, Q. No. 3 and Page No. 106, 'Meaning of Statistics'.

PART-A

Note: Answer the following questions.

Q. 2. Solve the following system of equations, using matrix method:

$$x = 2y + z = 7$$
$$x + 3z = 11$$
$$2x - 3y = 1$$

Ans. We have:

$$D = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 2 & -3 & 0 \end{vmatrix}$$

$$= 1\,(0 + 9) - 2\,(0 - 6) + 1\,(-3 - 0)$$
$$= 9 + 12 - 3 = 18$$

$\therefore \quad D = 18$

Since $D \neq 0$, hence solution is unique and can be obtained as:

$$x = \frac{D_1}{D};\ y = \frac{D_2}{D};\ z = \frac{D_3}{D}$$

$$D_1 = \begin{vmatrix} 7 & 2 & 1 \\ 11 & 0 & 3 \\ 1 & -3 & 0 \end{vmatrix}$$

$$= 7\,(0 - 9) - 2\,(0 - 3) + 1(-33 - 0)$$
$$= -63 + 6 - 33$$
$$= -90$$

$$D_2 = \begin{vmatrix} 1 & 7 & 1 \\ 1 & 11 & 3 \\ 2 & 1 & 0 \end{vmatrix}$$

$$= 1\,(0 - 3) - 7\,(0 - 6) + 1\,(1 - 22)$$
$$= -3 + 42 - 21$$
$$= 18$$

$$D_3 = \begin{vmatrix} 1 & 2 & 7 \\ 1 & 0 & 11 \\ 2 & -3 & 1 \end{vmatrix}$$

$$= 1\,(0 + 33) - 2\,(1 - 22) + 7\,(-3 - 0)$$
$$= 33 + 42 - 21$$
$$= 54$$

So,

$$x = \frac{D_1}{D} = \frac{-90}{18} = -5$$

$$y = \frac{D_2}{D} = \frac{18}{18} = 1$$

$$z = \frac{D_3}{D} = \frac{54}{18} = 3$$

$x = -5;\ y = 1;\ z = 3.$

Q. 3. Find $\frac{dy}{dx}$ for any *four* from the following:

(a) $y = u^4$ **and** $u = (x^2 + x + 1)$

(b) $x^2 + y^2 = 2xy$

QUESTION PAPER

December – 2022

(Solved)

BUSINESS MATHEMATICS AND STATISTICS

B.C.O.C.-134

Time: 3 Hours] ***[Maximum Marks: 100***

Note: *Question* ***No. 1*** *is compulsory. (ii) Answer both Part A and Part B. (iii) All questions carry* ***equal*** *marks.*

Q. 1. *(a)* Define Matrix. Discuss the types of matrices with an example.

Ans. Ref.: See Chapter-1, Page No. 1, 'Matrix' and 'Types of Matrices'.

***(b)* What is Dispersion? Discuss the significance of measuring dispersion.**

Ans. Ref.: See Chapter-14, Page No. 151, 'Concept of Dispersion' and 'Significance of Measuring Dispersion' and Page No. 165, Q. No. 1.

PART-A

(Business Mathematics)

Note: Attempt the following questions from this Part. All questions carry *equal* marks.

Q. 2. *(a)* find the matrices A and B from the following relation:

$$3A + B = \begin{bmatrix} 2 & \\ 8 & 9 \end{bmatrix}$$

and $$2A - B = \begin{bmatrix} 3 & 3 \\ 12 & 11 \end{bmatrix}.$$

Ans. *(a)* Given $3A + B = \begin{bmatrix} 2 & 0 \\ 8 & 9 \end{bmatrix}$ ...*(i)*

$$2A - B = \begin{bmatrix} 3 & 3 \\ 12 & 11 \end{bmatrix} \quad ...(ii)$$

Adding *(i)* and *(ii)*, we get

$$5A = \begin{bmatrix} 5 & 3 \\ 20 & 20 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3/5 \\ 4 & 4 \end{bmatrix}$$

Now

$$B = 2A - \begin{bmatrix} 3 & 3 \\ 12 & 11 \end{bmatrix}$$

$$= 2\begin{bmatrix} 1 & 3/5 \\ 4 & 4 \end{bmatrix} - \begin{bmatrix} 3 & 3 \\ 12 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 6/5 \\ 8 & 8 \end{bmatrix} - \begin{bmatrix} 3 & 3 \\ 12 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -9/5 \\ -4 & -3 \end{bmatrix}$$

Hence $A = \begin{bmatrix} 1 & 3/5 \\ 4 & 4 \end{bmatrix}$; $B = \begin{bmatrix} -1 & -9/5 \\ -4 & -3 \end{bmatrix}$.

***(b)* Find $A^2 + 3A - 2I$, where $A = \begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix}$.**

Ans. Given $A = \begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 24 \\ 40 & 64 \end{bmatrix}$$

$$3A = 3\begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 9 \\ 15 & 21 \end{bmatrix}$$

Sample Preview of The Chapter

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BUSINESS MATHEMATICS AND STATISTICS

PART A : BUSINESS MATHEMATICS

Introduction to Matrices

1

INTRODUCTION

An arrangement of numbers into rows and columns is referred as matrix. It's features are: *(i)* Compact notation for describing sets of data. *(ii)* Efficient method for manipulating data sets. Due to these features it proved to be a handy tool for solving problems which can be presented in linear equation system. It is widely used in Engineering, Economics and Business, Sociology, Statistics, Physics, Medicine and Information Technology.

CHAPTER AT A GLANCE

MATRIX

Definition: Matrix is an arrangement of numbers into rows and columns enclosed by a pair of brackets viz, [ ] or ( ). For example, the following matrix as:

$$\begin{bmatrix} 10 & 42 & 45 & 11 \\ 12 & 17 & 62 & 27 \\ 21 & 16 & 78 & 29 \end{bmatrix}$$

The number of rows and the number of columns in a matrix together define the dimensions of the matrix or its order. To assign a dimension to the matrix always starts indicating the numbers of rows and then the number of columns. Since the above matrix has 3 rows and 4 columns, we say that its dimension is 3 × 4. The individual numbers in a matrix are called elements.

The element 42 identifies that elements on the intersection of the first row and second column. Rows runs horizontally and columns runs vertically.

A matrix is denoted by a bold capital letter and the elements within the matrix are denoted by lower case letters for example, an element represented as a_{23} in a matrix its to be position at the row and 3rd column.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{1n} \\ a_{21} & a_{22} & a_{23} & a_{2n} \\ a_{31} & a_{32} & a_{33} & a_{3n} \\ a_{m_1} & a_{m_2} & a_{m_3} & a_{mn} \end{bmatrix}$$

The above matrix can also be written as:

Where I = 1,2,3...m

J = 1,2,3...n

Indicating a $m \times n$ order matrix.

Types of Matrices

Now, we will address commonly used matrix in business.

(1) Rows Matrix: A matrix having a single row called row matrix. **Example:** [2 3 5]

(2) Columns Matrix: A matrix having only one column is called column matrix. **Example:** $\begin{bmatrix} 3 \\ 7 \\ 1 \end{bmatrix}$

(3) Rectangular Matrix: In this matrix, the arrangement of elements represents a rectangle shape.

Example: $\begin{bmatrix} 4 & 5 & 1 \\ 6 & 7 & 2 \end{bmatrix}$

(4) Square Matrix: A special kind of matrix that has as many rows across as it has columns up and down (the matrix of order $m \times n$ is a square matrix).

Example: $\begin{bmatrix} 3 & 4 & -2 & 2 \\ 8 & 0 & 4 & -3 \\ 2 & 5 & 6 & 9 \\ -1 & 2 & 1 & -2 \end{bmatrix}$

(5) Diagonal Matrix: A square matrix which consists of all zeroes off the main diagonal is called a diagonal matrix.

Square Matrix A = $[a_{ij}]$ is a diagonal matrix if $a_{ij} = 0$ for all $i = j$.

Example: $\begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$

(6) Scalar Matrix: A diagonal matrix is called a scalar matrix if all the diagonal entries are same and non-zero.

Example: $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

(7) Identity Matrix (Unit Matrix): The identity matrix (or unit matrix) is a diagonal matrix with all diagonal entries equal to 1. 'I' represents the identity matrix.

Example: $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(8) Triangular Matrix: A square matrix in which all the entries above the main diagonal are zero is called lower triangle and a square matrix in which all the entries below the main diagonal are zero is called upper triangular. A matrix that is either upper triangular or lower triangular is called triangular matrix.

***(i)* Lower Triangular Matrix**

Example: $\begin{bmatrix} 3 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 6 & 7 & -1 & 0 \\ -4 & 3 & 2 & 6 \end{bmatrix}$

***(ii)* Upper Triangular Matrix**

$$\begin{bmatrix} 3 & 2 & 1 & 6 \\ 0 & 7 & -3 & 8 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 8 \end{bmatrix}$$

(9) Null or Zero Matrix: A null or zero matrix is that matrix where all elements are 0. It is denoted by O.

Example: $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(10) Symmetric Matrix: A symmetric matrix is a square matrix that is equal to its transpose square matrix.

Example: $\begin{bmatrix} 0 & 1 & -2 \\ 1 & 2 & 0 \\ -2 & 0 & 5 \end{bmatrix}$

(11) Sub Matrix: A matrix obtained by deleting some of the rows and columns of matrix is said to be a sub-matrix.

Example: $\begin{bmatrix} 3 & 2 \\ 8 & 1 \\ -1 & 9 \end{bmatrix}$ and $\begin{bmatrix} 3 & 6 \\ -1 & 2 \end{bmatrix}$ are the sub-matrix

of $\begin{bmatrix} 3 & 2 & 6 \\ 8 & 1 & -1 \\ -1 & 9 & 2 \end{bmatrix}$

MATRIX ALGEBRA

We will learn about basic function of matrix in this chapter. Let us first define the equality after that we will examine the matrix algebra. The matrix algebra method have to be done in ordered manner. It may be helpful for solving the problem in comfort manner such as addition and multiplication.

Equality of Matrices

Two matrices are equal if they have the same dimensions and all corresponding elements are equal.

Example: Consider the two matrices given below:

$$A = \begin{bmatrix} -3 & 4 \\ x & y \end{bmatrix} \text{ and } B = \begin{bmatrix} -3 & 4 \\ 4 & 2 \end{bmatrix}$$

If A = B, then $x = 4$ and $y = 2$.

Addition and Subtraction of Two Matrices

Matrices can be added or sub-stracted only when they are of same dimension (order) and resultant matrix will also have the same order.

For example: $A = \begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 4 \\ 5 & 2 \end{bmatrix}$

$$A + B = \begin{bmatrix} 8 & 7 \\ 1 & 7 \end{bmatrix}$$

$$A - B = \begin{bmatrix} -4 & -1 \\ -9 & 3 \end{bmatrix}$$

Negation of a Matrix: The negation of the matrix A is the matrix (–1). A written as (–A).

For example, $A = \begin{bmatrix} 2 & -7 & 6 \\ 6 & 1 & -1 \end{bmatrix}$ then

$-A = \begin{bmatrix} -2 & 7 & -6 \\ -6 & -1 & 1 \end{bmatrix}$ Therefore, two matrices A and B subtracted and demonstrated as the sum of A and the negation of matrix B.

$$A - B = A + (-B)$$

Multiplication of Matrix by a Scalar Quantity

To multiply a matrix by a number or in matrix algebra terminology, by a scalar is to multiply every element of that matrix by the given scalar.

For example, if $A = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -6 \end{bmatrix}$, then

$$3A = \begin{bmatrix} 9 & -3 & -6 \\ 6 & 0 & -18 \end{bmatrix}$$

Properties of Addition of Matrices

1. Addition of Matrices is Commutative: If A and B are two matrices of same order, then

$$A + B = B + A$$

2. Addition of Matrices is Associative: If A, B and C are three matrices of same order, then

$$(A + B) + C = A + (B + C)$$

3. Existence of Additive Identity: If A is a matrix and O is the null matrix of the same order as that of A, then

$$A + O = O + A = A$$

4. Existence of Additive Inverse : For any matrix,

$$A + (-A) = (-A) + A = O$$

Multiplication of Two Matrices

The multiplication or product of two matrices A and B is defined if the number of columns of A is equal to the number of rows of B. Let $A = [a_{ij}]$ be an $m \times n$ matrix and $B = [b_{jk}]$ be an $n \times p$ matrix. Then the product of A and B would be the matrix C of order $m \times p$.

For example: $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $B \begin{bmatrix} 10 & 11 \\ 20 & 21 \\ 30 & 31 \end{bmatrix}$

$A \times B = C =$

$$\begin{bmatrix} 1\times10+2\times20+3\times30 & 1\times11+2\times21+3\times31 \\ 4\times10+5\times20+6\times30 & 4\times11+5\times21+6\times31 \end{bmatrix}$$

$$= \begin{bmatrix} 10+40+90 & 11+42+93 \\ 40+100+180 & 44+105+186 \end{bmatrix}$$

$$= \begin{bmatrix} 140 & 146 \\ 320 & 335 \end{bmatrix}$$

Properties of Matrix Multiplication

1. Associativity: Matrix multiplication is associative. It means that for any three matrices A, B and C.

$$(AB)\,C = A\,(BC).$$

2. Distributive Over Addition: The distributive property of multiplication over addition can be used when you multiply a number by a sum. For example, A,B, and C of order $m \times p$, $n \times p$ and $p \times q$ respectively.

$$A\,(B + C) = AB + AC$$

3. Identity: Matrix A of order $m \times n$, there is an identity matrix. I_n of order $n \times n$ and an identity matrix I_m of order $m \times m$ such that $I_m A = A = AI_n$.

TRANSPOSE OF A MATRIX

The transpose of a matrix is simply a flipped version of the original matrix. We can transpose the matrix by switching its row with its columns. For example, matrix $A = [a_{ij}]$ order $m \times n$ we switched its row and column.

$A = \begin{bmatrix} 1 & 4 \\ 0 & 7 \\ 3 & -2 \end{bmatrix}$ then $A' = \begin{bmatrix} 1 & 0 & 3 \\ 4 & 7 & -2 \end{bmatrix}$

The matrix consequently obtained is called transpose of A and denoted by A'.

Properties of Transpose of a Matrix

(i) $(A')' = A$

(ii) $(kA)' = kA'$ where k is some scalar quantity.

(iii) $(A+B)' = A' + B'$

(iv) $I' = I$

(v) $(AB)' = B'\,A'$

1. Symmetric Matrix

Matrix A is called symmetric matrix if $A' = A$.

For example, if:

$$A = \begin{bmatrix} 2 & 6 & 5 \\ 6 & -2 & 7 \\ 5 & 7 & 3 \end{bmatrix}$$

So A is a symmetric matrix.

2. Skew Symmetric Matrix

Matrix A is called skew symmetric matrix if $A' = -A$. **For example,** if:

$$A = \begin{bmatrix} 0 & 1 & -8 \\ -1 & 0 & -2 \\ 8 & 2 & 0 \end{bmatrix}$$