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Based on

N.I.O.S. *Class* – XII

National Institute of Open Schooling

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CONTENTS

MATHEMATICS

Based on: ***NATIONAL INSTITUTE OF OPEN SCHOOLING – XII***

S.No.	*Chapters*	*Page*

■ ■

Sample Preview
of the
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Sample Question
Papers

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Solved Sample Paper - 1

Based on NIOS (National Institute of Open Schooling)

Mathematics - XII

Time : 3 Hours] **[Maximum Marks : 100**

Note : *(i)* This question paper consists of **four sections** A, B, C and D containing **33** questions.

(ii) Question Number **1 to 10** in **Section A** are Multiple Choice Questions (MCQ). Each question carries one mark. In each question there are four choices (A, (B), (C) and (D) of which only one is correct. You have to select the correct choice and indicate it in your answer book by writing (A), (B), (C) or (D) as the case may be. No seperate time its allotted for attempting MCQs.

(iii) Question Number **11 to 16** in **Section B** are very short answer questions and carry **2 marks** each.

(iv) Question Number **17 to 28** in **Section C** are short answer questions and carry **4 marks** each.

(v) Question Number **29 to 33** in **Section D** are long answer questions and carry **6 marks** each.

(vi) All questions are **compulsory.** There is no overall choice, however, alternative choices are given in some questions. In such questions, you have to attempt only one choice.

SECTION - A

Q. 1. The matrix A = $\begin{bmatrix} 0 & 0 & 3 \\ 0 & 3 & 0 \\ 3 & 0 & 0 \end{bmatrix}$ **is a:**

(a) **Scalar matrix** *(b)* **Diagonal matrix**
(c) **Unit matrix** *(d)* **Square matrix**

Ans. *(a)* Scalar matrix.

Q. 2. If A = {a, b}, then the number of binary operations that can be defined on A:

(a) **16** *(b)* **4**
(c) **2** *(d)* **1**

Ans. *(a)* 16.

Q. 3. The principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ **is:**

(a) $-\frac{\pi}{6}$ *(b)* $-\frac{\pi}{3}$
(c) $\frac{5\pi}{6}$ *(d)* $\frac{2\pi}{3}$

Ans. *(d)* $\frac{2\pi}{3}$.

Q. 4. The degree of the differential equation $\left(\frac{d^3y}{dx^2}\right) - \left(\frac{dy}{dx}\right)^3 - \sin^2 x = 0$ **is:**

(a) **3** *(b)* **2**
(c) **1** *(d)* **not defined**

Ans. *(b)* 2.

Q. 5. $\int e^x (\sin x + \cos x)\, dx$ **is equal to:**

(a) $e^x \cos x + c$ *(b)* $-e^x \cos x + c$
(c) $e^x \sin x + c$ *(d)* $-e^x \sin x + c$

Ans. *(c)* $e^x \sin x + c$.

Q. 6. $\int \tan x\, dx$ **is equal to:**

(a) $\sec^2 x + c$ *(b)* $\sec x \tan x + c$
(c) $\log|\sec x| + c$ *(d)* $\log|\cos x| + c$

Ans. *(c)* $\log|\sec x| + c$.

Q. 7. $\lim_{x \to 0} \frac{\sin 5x}{x}$ **is equal to:**

(a) **1** *(b)* $\frac{1}{5}$
(c) **5** *(d)* **0**

Ans. *(c)* 5.

Q. 8. The differential coefficient of sec $(\tan^{-1} x)$ **with respect to** x **is:**

(a) $\frac{x}{1+x^2}$

(b) $\frac{x}{\sqrt{1+x^2}}\vec{b} = \hat{j} - \hat{k} \; -\frac{\pi}{6}$

(c) $x\sqrt{1+x^2}$

(d) $\frac{1}{\sqrt{1+x^2}}$

Ans. *(c)* $\frac{x}{\sqrt{1+x^2}}$.

Q. 9. The angle between the vectors $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = \hat{j} - \hat{k}$ is:

(a) $-\frac{\pi}{3}$ *(b)* $\frac{\pi}{3}$

(c) $\frac{2\pi}{3}$ *(d)* $\frac{\pi}{6}$

Ans. *(b)* $\frac{\pi}{3}$

Q. 10. Which of the following sentences is *not a* statement?

(a) **The sun is a star**
(b) **Lahore is in India**
(c) **Every rectangle is a square**
(d) **Mathematics is a fun**

Ans. *(d)* Mathematics is a fun.

SECTION – B

Q. 11. If A = $\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$, find 3A – B.

Ans. $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

$$\Rightarrow \quad 3A - B = 3\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times 3 & 4\times 3 \\ 3\times 3 & 2\times 3 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 6-(-2) & 12-5 \\ 9-3 & 6-4 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$$

Or

If $\begin{bmatrix} 1 & 2 & -3 \\ 0 & 4 & 1 \end{bmatrix} + A = \begin{bmatrix} 3 & 5 & 6 \\ -1 & 0 & 3 \end{bmatrix}$, find the matrix A.

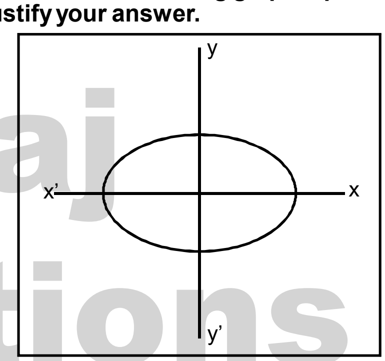
Ans. $\begin{bmatrix} 1 & 2 & -3 \\ 0 & 4 & 1 \end{bmatrix} + A = \begin{bmatrix} 3 & 5 & 6 \\ -1 & 0 & 3 \end{bmatrix}$

$$A = \begin{bmatrix} 3 & 5 & 6 \\ -1 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 2 & -3 \\ 0 & 4 & 1 \end{bmatrix}$$

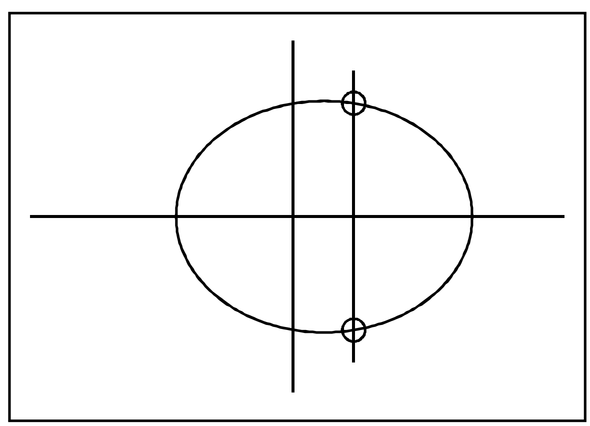
$$= \begin{bmatrix} 3-1 & 5-2 & 6-(-3) \\ -1-0 & 0-4 & 3-1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & 9 \\ -1 & -4 & 2 \end{bmatrix} \text{ Ans.}$$

Q. 12. Does the following graph represent a function? Justify your answer.



Ans. Functions need to be well-defined as part of their definition, so for a given input there can only be one output. In the given graph if we draw a vertical line, we can have the two values, thus it is not a function.



Q. 13. If $y = e^x \cos x$, find $\frac{d^2y}{dx^2}$.

Ans. $y = e^x \cos x$

Sample Preview
of
The Chapter

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MATHEMATICS

MODULE-I: SETS, RELATIONS AND FUNCTIONS

Sets

1

CHAPTER AT A GLANCE

Definition: A set is a well defined collection of distinguishable objects.

A collection may or may not be a set. A collection will not be a set if objects cannot be distinguished, for example: A collection of most loving parents of a particular locality is not a set as measurement of love is not possible. Similarly, there are many things that can not be measured. Their collection is not a set. Only a collection whose elements are distinguishable can be defined as a set.

So, we give a list as follows:

A collection not a set	*A collection which is set*
1. Rich people of Delhi.	1. People with more than Rs. 1 Lac in their bank.
2. Good mannered students of class XII.	2. Students of class XII who daily say, "Good Morning" to their Principal.
3. Interesting books written by Prem Chand.	3. Collection of books written by Prem Chand.
4. Hard working students of class XI.	4. Students of class XI who study at least 4 hours daily at home.

Some Notation in set theory:

N : The set of natural numbers.
W : The set of whole numbers.
Z or I : The set of integers.
Z^+ : The set of positive integers.
Z^- : The set of negative integers.
Q : The set of rational numbers.
R : The set of real numbers.
C : The set of complex numbers.
$\in$: Belongs to.
$\notin$: does not belong to.
$\exists$: There exists.
$\subset$: Subset of.
$\not\subset$: not a subset of.
$\supset$: super set.
$\not\supset$: not a super set.
$\nexists$: does not exist.

How to Represent a Set?

1. Roaster Form or Tabular Form : Listing the individual elements/members in paranthes is roaster form. For example:

$A = \{1, 2, 3, \text{...........}, 10\}$
$B = \{\text{Elephant, Horse, Donkey}\}$
$C = \{a, e, i, o, u\}$

2. Set Builder Form: When we define a property of each element of a set, we have set builder form. For example:

$A = \{x : x \in N, x \leq 10\}$
$B = \{x : x \text{ is elephant, or horse or donkey}\}$
$C = \{x : x \text{ is a vowel of English alphabet}\}$

Type of Sets

1. Finite Set: If the number of element in a set is countable, then it is called a finite set.

e.g. $A = \{1, 2, \text{................} 1000\}$
$B = \{2, 4, \text{........}, 2n, n<1 \text{ million } n \in N\}$

A, B are finite sets.

2. Infinite Set: A set which has uncountable elements is called an infinite set

e.g. $A = \{1, 2, 3, \text{...........}\}$
$B = \{\text{........}, -3, -2, -1, 0, 1, 2\}$

A, B are infinite sets.

3. Empty Set: The set with no element is called an empty set. It is written as { } or ϕ., i.e.,

$\phi = \{\ \}$

We must note that 0, {0} or {ϕ} are not empty sets.

4. Singleton Set: A set with only one element is called a singleton, e.g.,

A = {a}
B = {x : x is an even prime number}

A, B are singleton.

5. Equal Sets: Two or more sets are called equal sets if they contain exactly same elements.

e.g. A = {2},
B = {x : x is an even prime number}
∴ A = B
or {1, 2, 3} = {3, 2, 1}

6. Equivalent Sets: Two sets having equal number of elements (they may have different elements) are called equivalent sets.

e.g. {1, 2, 3} and {a, b, c} are equivalent sets.

7. Disjoint Sets: Two sets that have no element in common are called disjoint sets.

e.g. {1, 2, 3}, {5, 6, 7} are disjoint sets.

Subset

Definition: Let A and B be two sets such that every element of A is also an element of B, then A is subset of B, written as, $A \subseteq B$.

In case B has at least one element more than B then A is called a proper subset of B and B is called the superset of A, i.e., $A \subset B$ and $B \supset A$.

We note if $A \subset B$ and $B \subset A$

$\Rightarrow$ A = B

Also empty set (null or void set) ϕ is subset of every set A, i.e. $\phi \subset A$.

Power Set

Definition: A set of all subsets of a set A is called the power set of A.

Let A = {p, q}

then all possible subsets of A, viz.,

ϕ, {p}, {q}, {p, q} written in set form, i.e.

P(A) = {ϕ, {p}, {q}, {p, q}} is the power set of A.

Universal Set

Definition: If a set ∪ is superset of all sets under consideration, then ∪ is called the Universal Set.

Another definition could be as follows:

If every set under consideration, say A, B, C, D etc. are subsets of a set ∪ then ∪ is called the Universal for these sets A, B, C, D ....., etc.

Venn Diagram

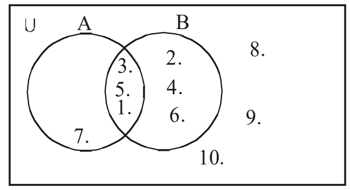
Sets could be represented by diagrams, known as Venn diagram, so called in honour of introducer of this method, The British Mathematician John Venn.

A Universal set ∪ can be represented in a rectangular form and all other sets which will be subsets of ∪, by circles.

For example, if

A = {1, 3, 5, 7},
B = {1, 2, 3, 4, 5, 6}
U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

Then Venn diagram is as follows:



Difference of Sets: Let A and B be any two sets then their difference A–B is defined as the sets of elements of A not common with elements of B.

In fact, we first return back to Universal Set with a very interesting discussion as follows:

We know the set ∪ whose subsets are all sets under consideration is called Universal set. Let me ask the following question:

"Is there a 'Universal Set?" What I mean by this is – Is there a Universal set which contains every possible objects (or even ideas)?

Well let such a set, say N is possible, i.e., N = set of all objects possible in the world. ...(1)

Then obviously $N \subseteq N$. But let us ask a very relevant question, viz.

Does $N \in N$?

There are only two possibilities – either $N \in N$ or $N \notin N$, and third possibility, that is, $N \in N$ and $N \notin N$ is totally ruled out. Reason is clear that such a confusing situation makes the collection N not a set, whereas we have taken N as a set.

Now we discuss above two possiblities one by one

Case I: Let $N \in N$

$\Rightarrow$ N is an element of set N.

$\Rightarrow$ N is not a set.

$\Rightarrow$ N Ï N.

Case II: Lert N Ï N.

$\Rightarrow$ N is one element in the world which is not in N and So the definition of N as in (1) is violated.

$\Rightarrow$ $N \in N$.

So we have

(i) $N \in N \Rightarrow$ N Ï N

(ii) N Ï N $\Rightarrow N \in N$

So *(i)* equation and *(ii)* are totally absurd conclusions of above discussion. How about it? What is wrong with this? Is there something wrong with the definition of N? Not really except that N is different type of set all together. In fact N is an abnormal set. All other sets are normal sets. Only this particular one to above analogy. So we can say in normal sense, there is no Universal set of elements of all Universal sets.

Here is a parallel discussion of above in a different setting:

Let us have a locality A which has a barber B who shaves all people of A. No other barber does visit A and nor does a person of A go outside for the purpose of shave.

Now B must be shaved too. We have a basic rule in the words of B as B saying.

"I shave those and only those who do not shave themselves."

Question is–"Does B shave himself ?"

Obviously answer can be either yes or no, both yes and no are not possible in view of 'those and only those, condition contained in what B says.

Now let us take each of the possible answers one by one.

Case I : Yes $\Rightarrow$ B shaves himself.

But B is the barber who does not shave anyone except 'non-self-shavers' if I can use the words for 'those who do not shave themselves'. So, B being a 'self shaver' cannot shave himself as a barber.

i.e., Yes $\Rightarrow$ No, he cannot shave himself.

Case II: No $\Rightarrow$ He is a 'non-self shaver' and since as a barber 'B' has got to shave 'B' as a person.

$\Rightarrow$ If barber B does not shave B as a person, he has to shave B.

i.e., No $\Rightarrow$ Yes, he has to shave himself.

So paradoxically, Yes $\Rightarrow$ No and No $\Rightarrow$ Yes.

Hence our conclusion is:

"There is no Universal 'Universal Set' and any Universal set is specific to a particular situation only."

Now we are in a position to define what is called the complement of a set.

Let U = Universal Set

and A = A set such that $A \subseteq U$,

then complement of A written as A' or A^c is, -

$$A' = U - A$$

Let us explain the complement of a set by an example. Let U = Set of Natural numbers

A = Set of even Natural numbers

then $A' = U - A$

= Set of odd natural numbers

It is very clear only odd and even natural numbers exist so;

Complement of even natural numbers = Set of odd natural numbers and

Complement of odd natural numbers = set of even natural numbers.

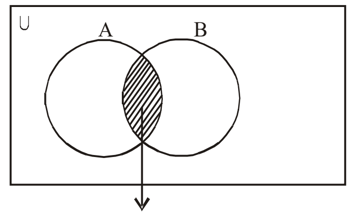
Let us take the following questions and solve them to explain difference and complement of a set by Venn diagram and Roaster Set builder form.

Intersection of Sets

Definition: Let A, B be two sets, then their intersection written as $A \cap B$ is defined as the set of common elements of elements of A and B, i.e.,

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

In Venn diagram, the same could be depicted by shaded region as shown below:

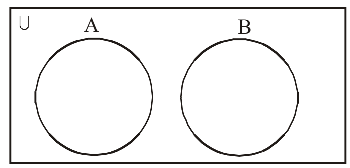


$A \cap B$

Disjoint Sets: Two sets A and B are said to be the disjoint sets if they have no element in common, i.e.,

$$A \cap B = \phi.$$

Venn Diagram for disjoint sets, A, B is given below:



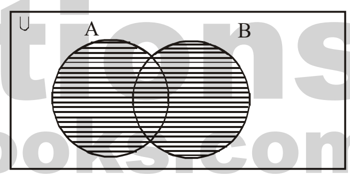
A, B are disjoined $\Rightarrow A \cap B = \phi$.

Union of Sets

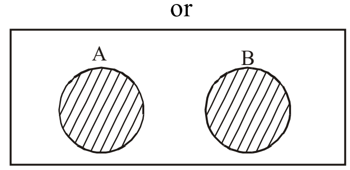
Definition: If A, B be two sets then their union, written as $A \cup B$ is the set of elements of either set A or set B or their intersection, i.e.,

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

As repetition of elements in a set is of no use and their order of writing too does not matter, So by Venn Diagram, we can have $A \cup B$ as follows:



or



CHECK YOUR PROGRESS 1.1

Q. 1. Which of the following collections are sets?

(i) **The collection of days in a week starting with S.**

(ii) **The collection of natural numbers upto fifty.**

(iii) **The collection of poems written by Tulsidas.**

(iv) **The collection of fat students of your school.**

Ans. *(i)* The collection of days in a week starting with S = {Sunday, Saturday} is a set.

(ii) The collection of natural numbers upto fifty = {1, 2, 3, ..., 50} is a set.

(iii) Collection of poems written by Tulsidas is a set.

(iv) The collection of fat students of your school is not a set as fatness cannot be measured.

Q. 2. Insert the appropriate symbol in blank spaces.

If A = {1, 2, 3}.

(i) **1...........A**

(ii) **4........A.**

Ans. Given A = {1, 2, 3}

(i) 1 ....∈.... A

(ii) 4 ....∈... A

Q. 3. Write each of the following sets in the Roster form:

(i) **A = {x :$x \in z$ and – 5 ≤ x ≤ 0 }.**

(ii) **B = {x : $x \in$ R and $x^2 - 1 = 0$}.**

(iii) **C = {x : x is a letter of the word banana}.**

(iv) **D = {x : x is a prime number and exact divisor of 60}.**

Sol. *(i)* A = {x :$x \in z$ and – 5 ≤ x ≤ 0 }
= {– 5, – 4, – 3, – 2, – 1, 0}

(ii) B = {x : $x \in$ R and $x^2 - 1 = 0$}
= {1, – 1}

(iii) C = {x : x is a letter of the word banana}
= {b, a, n}

(iv) D = {x : x is a prime number and exact divisor of 60}
= {2, 3, 5}.

Q. 4. Write each of the following sets in the set builder form:

(i) **A = {2, 4, 6, 8, 10}**

(ii) **B = {3, 6, 9,...... ∞ }**

(iii) **C = {2, 3, 5, 7}**

(iv) **D = $\left\{-\sqrt{2}, \sqrt{2}\right\}$**

Are A and B disjoints sets?

Sol. *(i)* A = {2, 4, 6, 8, 10}
= {x : $x = 2y$, $y \in$ N, $y \le 5$}

(ii) B = {3, 6, 9,...... ∞ }
= {x : $x = 3y$, $y \in$ N}

(iii) C = {2, 3, 5, 7}
= {x : x is a prime number ≤ 7}

(iv) D = $\left\{-\sqrt{2}, \sqrt{2}\right\}$
= {x : $x \in$ R and $x^2 - 2 = 0$}

As 6 ∈ A, 6 ∈ B

So A, B are not disjoint.

Q. 5. Which of the following sets are finite and which are infinite?

(i) **Set of lines which are parallel to a given line.**

(ii) **Set of animals on the earth.**

(iii) **Set of Natural numbers less than or equal to fifty.**

(iv) **Set of points on a circle.**

Ans. *(i)* Infinite; *(ii)* Finite ; *(iii)* Finite; *(iv)* Infinite.

Q. 6. Which of the following are null set or singleton?

(i) **A = {x : $x \in$ R and x is a solution of $x^2 + 2 = 0$}.**

(ii) **B = {x : $x \in$ Z and x is a solution of $x - 3 = 0$}.**

(iii) **C = {x : $x \in$ Z and x is a solution of $x^2 - 2 = 0$}.**

(iv) **D = {x : x is a student of your school studying in both the classes XI and XII }**

Sol. *(i)* A = {x : $x \in$ R and x is a solution of $x^2 + 2 = 0$}
= {x : $x \in$ R, $x = \pm\sqrt{2}$ }
= φ, as $\sqrt{-2}$ is not a real number.

And so $-\sqrt{-2}$ is not a real number.

∴ A is a null set.

(ii) B = {x : $x \in$ Z and x is a solution of $x - 3 = 0$}
= {x : $x \in$ Z, and $x = 3$ }

∴ B is a singleton.

(iii) C = {x : $x \in$ Z and x is a solution of $x^2 - 2 = 0$}
= {x : $x \in$ Z, and $x = \pm\sqrt{2}$ }= φ

∴ C is a null set.

(iv) D = {x : x is a student of your school studying in both the classes XI and XII }
= φ

∴ D is a null set.

Q. 7. In the following check whether A = B or A » B ·

(i) **A = {a}, B = {x : x is an even prime number}.**

(ii) **A = {1, 2, 3, 4}, B = {x : x is a letter of the word guava}.**

(iii) **A = {x : x is a solution of $x^2 - 5x + 6 = 0$ }, B = {2 , 3}.**

Sol. *(i)* A = {a}
B = {x : x is an even prime number}

∴ B = {2}

∴ A, B are equivalent sets.