Serving Education for Last 40 Years

NEERAJ®

M.E.C.-103

Quantitative Methods

Chapter Wise Reference Book
Including Many Solved Sample Papers

Based on

I.G.N.O.U.

& Various Central, State & Other Open Universities

By: *Pritilata,* M.B.A

NEERAJ PUBLICATIONS
(Publishers of Educational Books)

Mob.: 8510009872, 8510009878 E-mail: info@neerajbooks.com
Website: www.neerajbooks.com

MRP ₹ 380/-

Content

QUANTITATIVE METHODS

S.No.	Chapterwise Reference Book	Page

■■

Sample Preview of the Solved Sample Question Papers

Published by:

NEERAJ PUBLICATIONS
www.neerajbooks.com

QUESTION PAPER

June – 2023

(Solved)

QUANTITATIVE METHODS

M.E.C.-103

Time: 3 Hours] *[Maximum Marks: 100*

Note: *Answer the questions from each section as directed.*

SECTION-A

Note: Answer any *two* questions from this Section.

Q. 1. *(a)* The production function of a firm that uses only one variable input (labour) is given by:

$$x = 125L + L^2 - 0.1L^3$$

Find out the marginal cost, if the firm employees 20 units of labour and the wage rate is fixed at Rs. 90 per unit.

Ans. The total cost function (TC) is the product of the wage rate (W) and the quantity of labor (L):

$$TC = W \times L$$

In this case, the wage rate is given as Rs. 90 per unit of labour, so W = 90.

The quantity of labour (L) is given as 20 units.

Substituting the given values into the production function $x = 125L + L^2 + 0.1L^3$, we can find the total output *(x)* when 20 units of labour are employed:

$$x = 125(20) + (20)^2 - 0.1(20)^3$$
$$x = 2500 + 400 - 80$$
$$x = 2420$$

So, when the firm employs 20 units of labour, the total output *(x)* is 2420 units.

Now, the total cost (TC) is given by TC = W × L= 90 × 20 = 1800.

The marginal cost (MC) is the derivative of the total cost function with respect to the quantity of output *(x)*:

$$MC = dTC/dx$$

Differentiating the total cost function TC=1800 with respect to x, we get:

$$MC = 0$$

Therefore, the marginal cost of production, when the firm employs 20 units of labour and the wage rate is fixed at Rs. 90 per unit, is zero.

***(b)* Prove that if *f′ (a)* is finite, *f(x)* must be continuous.**

Ans. Ref.: See Chapter-2, Page No. 11, Q. No. 1

Q. 2. Max.:

$$U(z_1, z_2) = \frac{z_1}{1+z_1} + \frac{z_2}{1+z_2}$$

Subject to:

$$z_1 \geq 0$$
$$z_2 \geq 0$$
$$P_1z_1 + P_2z_2 \geq 1.$$

Are the indifference curves downward sloping, convex, and cut the axes? Examine and justify your answer.

Ans. Downward Sloping Indifference Curves: To determine if the indifference curves are downward-sloping, we can calculate the marginal rate of substitution (MRS) between z_1 and z_2. The MRS represents the rate at which a consumer can give up some amount of one good (z_1) to obtain an additional unit of the other good (z_2) while maintaining the same level of utility.

The MRS is given by:

$$MRS = -\frac{\frac{\partial U}{\partial z_1}}{\frac{\partial U}{\partial z_2}}$$

Calculating the partial derivatives:

$$\frac{\partial U}{\partial z_1} = \frac{1}{(1+z_1)^2}$$

$$\frac{\partial U}{\partial z_2} = \frac{1}{(1+z_2)^2}$$

Therefore, the MRS is:

$$MRS = \frac{\frac{1}{(1+z_1)^2}}{\frac{1}{(1+z_2)^2}} = -\frac{(1+z_2)^2}{(1+z_1)^2}$$

Since, the MRS is negative, the indifference curves are downward-sloping.

2. Convexity of Indifference Curves: To check if the indifference curves are convex, we need to examine the second-order partial derivatives. If the function is concave up (second-order partial derivatives are positive), the indifference curves are convex.

$$\frac{\partial^2 U}{\partial z_1^2} = \frac{2}{(1+z_1)^3}$$

$$\frac{\partial^2 U}{\partial z_1^2} = \frac{2}{(1+z_2)^3}$$

Both second-order partial derivatives are positive, indicating convexity.

3. Indifference Curves Cutting the Axes: To find points where the indifference curves cut the axes, set z_1 or z_2 to 0 and solve for the other variable. For example:

1. Cutting the z_1-Axis:

$$U(0, z_2) = \frac{0}{0+1} + \frac{z_2}{1+z_2} = \frac{z_2}{1+z_2}$$

The curve cuts the z_1-axis when $z_2 = 0$.

2. Cutting the z_2-Axis:

$$U(z_1, 0) = \frac{z_1}{1+z_1} + \frac{0}{0+1} = \frac{z_1}{1+z_1}$$

The curve cuts the z_2-axis when $z_1 = 0$.

Both cases show that the indifference curves cut the axes.

In summary, the given utility function represents downward-sloping, convex indifference curves that cut both axes. This utility function conforms to typical preferences and satisfies the standard assumptions about consumer behavior.

Q. 3. If P = *f (x)* is an inverse demand function, find out the level of output at which total revenue is maximum. Show that total revenue will always be a maximum if the demand curve is downward sloping and concave from below. Is it possible to have maximum total revenue if the demand curve is convex from below? Discuss.

Ans. We first need to understand the relationship between total revenue (TR) and quantity *(x)* given an inverse demand function $p = f(x)$.

Total Revenue (TR) is calculated by multiplying the price *(p)* at which each unit is sold by the quantity *(x)* of units sold. Mathematically, TR is given by:

$$TR = p \times x$$

Now, if $p = f(x)$, we can substitute this expression for p into the equation for TR:

$$TR = f(x) \times x$$

To find the level of output *(x)* at which TR is maximum, we can differentiate TR with respect to x and set the derivative equal to zero to find the critical points. We then use the second derivative test to determine if these points correspond to a maximum.

Now, regarding the second part of your question: total revenue will always be maximized if the demand curve is downward-sloping and concave from below. Here's why:

Downward-Sloping Demand Curve: A downward-sloping demand curve means that as the quantity sold increases, the price per unit decreases. This characteristic ensures that increasing production and sales generally lead to higher total revenue. If demand is upward-sloping, total revenue could be maximized at low levels of output, which is not desirable.

Concave from Below: Concavity from below (meaning the demand curve is curved upwards) implies diminishing marginal revenue. When marginal revenue becomes zero (where MR = 0), total revenue reaches its maximum point. This happens where the demand curve is tangential to the x-axis.

However, if the demand curve is convex from below (meaning it curves downward), there could be multiple critical points where marginal revenue is zero, leading to multiple maximum points for total revenue. This situation can arise in cases where the demand curve behaves in unusual ways, such as with Giffen goods or other complex market dynamics.

In summary, for most common goods and services, total revenue is maximized when the demand curve is downward-sloping and concave from below. Convexity from below can lead to complex scenarios where multiple maximum points exist, making it more challenging to determine the optimal level of output.

Q. 4. Suppose that the numbers x_1 and x_2 satisfy the equations $x_1 - 2x_2 = 3$ and $3x_1 + 5x_2 = 20$.

Find x_1 and x_2 by using the Cramer's rule.

Ans. The given system of equation is:

$$x_1 - 2x_2 = 3$$

$$3x_1 + 5x_2 = 20$$

$$\therefore \quad D = \begin{bmatrix} 1 & -2 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 20 \end{bmatrix}$$

$$x = \frac{D_1}{D} = \frac{34}{-20} = 1.7$$

$$y = \frac{D_2}{D} = \frac{-20}{20} = 1.$$

Sample Preview
of
The Chapter

Published by:

NEERAJ
PUBLICATIONS
www.neerajbooks.com

QUANTITATIVE TECHNIQUES

INTRODUCTION TO DIFFERENTIAL CALCULUS

Functions, Limit and Continuity

1

INTRODUCTION

Mathematical Techniques are very useful in order to solve economic problems. Some concepts are related to function, limit, and continuity are expressed here. There are many ways to describe or represent functions: by a formula, by an algorithm that computes it, or by plotting graph. Making relationship between describing the problem and then interpreting it, is the main function of these mathematical expressions. In order to clear the concepts we will discuss some illustrations and also some practical exercises. The explanations given here will manage the understanding of different mathematical and statistical knowledge levels of students in each and every aspect.

CHAPTER AT A GLANCE

REVIEW OF THE BASIC CONCEPTS

Set

Sets are "collections". The objects "in" the collection are its members, e.g. we are all members of the set of all humans. There are sets of numbers, people, and other sets.

Example: $\{x : x \text{ is an even number}\}$

The set containing the even numbers (i.e. {0, 2, 4, ...})

FUNCTION

It is defined as the one quantity (the argument of the function, also known as the input) that completely determines another quantity (the value, or the output).

Domain: The set of all permitted inputs to a given function is called the domain of the function. Range: The set of all resulting outputs is called the image or range of the function.

Co-domain: The image is often a subset of some larger set, called the co-domain of a function. For example, the function $f(x) = x^2$ could take as its domain the set of all real numbers, as its image the set of all non-negative real numbers, and as its co-domain the set of all real numbers.

A function assigns exactly one value to each input of a specified type. The argument and the value may be real numbers, but they can also be elements from any given sets: the domain and the co-domain of the function.

Example of a function with the real numbers as both its domain and co-domain:

Function $f(x) = 2x$, which assigns to every real number the real number with twice its value.

In this case, it is written that $f(5) = 10$.

The notation $f : X \rightarrow Y$ indicates that f is a function with domain X and co-domain Y.

Variable: These are numerical values which changes with different mathematical operations. For example, *a, b, x, y*……

Continuous Variable: If y takes all prime numbers from a given number a to another given number b, then y is known as a continuous variable.

Interval: The value between two terminal values are known as interval. In the expression $(x + 3).(x + 5)$, the interval of x is –3 and –5.

Constant

The numerical value which never changes with change in mathematical operation. i.e. 5, 6, ½ π. . .

Absolute Value: The absolute value of a number is its distance from 0 on a number line.

For example, the number 9 is 9 units away from 0. Therefore its absolute value is 9.

www.neerajbooks.com

In case of negative number, the absolute value is also positive. The number – 4 is still 4 units away from 0. The absolute value of – 4 is therefore positive 4.

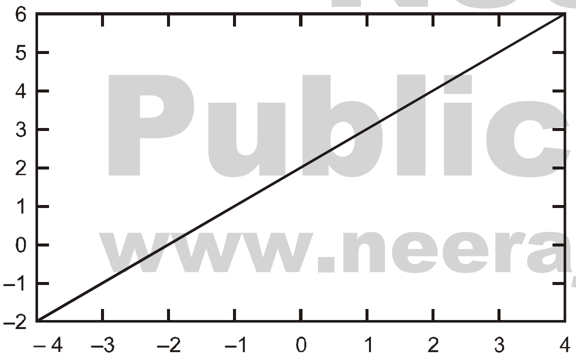
Examples:

$$|4| = 4$$
$$|-4| = 4$$
$$|4+3| = 7$$
$$|-4-3| = 7$$
$$|3-4| = 1$$
$$-|4| = -4$$
$$-|-4| = -4$$

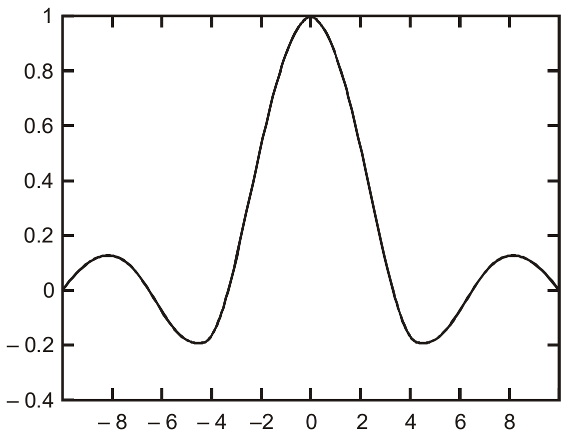
Graph of a Function

The symbol for the input to a function is often called the independent variable or argument and is often represented by the letter x or, if the input is a particular time, by the letter t. The symbol for the output is called the dependent variable or value and is often represented by the letter y. The function itself is most often called f, and thus the notation $y = f(x)$ indicates that a function named f has an input named x and an output named y.

Graph of the function $(x^2 - 4)/(x - 2)$



Graph of the function sin $(x)/x$



Bounded Functions and their Bounds

A function f defined on some set X with real or complex values is called bounded, if the set of its values is bounded. In other words, there exists a real number $M < \infty$ such that

$|f(x)| \leq M$ for all x in X.

Sometimes, if $f(x) \leq A$ for all x in X, then the function is said to be bounded above by A. On the other hand, if $f(x) \geq$ for all x in X, then the function is said to be bounded below by B.

Monotone Function

A monotonic function (or monotone function) is a function that preserves the given order. A function f defined on a subset of the real numbers with real values is called monotonic (also monotonically increasing, increasing or non-decreasing), if for all x and y such that $x \leq y$ one has $f(x) \leq f(y)$, so f preserves the order. Likewise, a function is called monotonically decreasing (also decreasing or non-increasing) if, whenever $x \leq y$, then $f(x) \leq f(y)$, so it reverses the order.

If the order $\leq$ in the definition of monotonicity is replaced by the strict order <, then one obtains a stronger requirement. A function with this property is called strictly increasing. Again, by inverting the order symbol, one finds a corresponding concept called strictly decreasing. Functions that are strictly increasing or decreasing are one-to-one (because for x not equal to y, either $x < y$ or $x > y$ and so, by monotonicity, either $f(x) < f(y)$ or $f(x) > f(y)$, thus $f(x)$ is not equal to $f(y)$).

Inverse Function

f is a function from a set A to a set B, then an inverse function for f is a function from B to A, with the property that a round trip (a composition) from A to B to A (or from B to A to B) returns each element of the initial set to itself. Thus, if an input x into the function f produces an output y, then inputting y into the inverse function produces the output x, and vice versa.

Types of Function

1. **Algebraic Function:** An algebraic function is informally a function that satisfies a polynomial equation whose coefficients are themselves polynomials. For example, an algebraic function in one variable x is a solution y for an equation where the coefficients $a_i\ (x)$ are polynomial functions of x.

   A constant function is a function whose values do not vary and thus are constant. For example,

Neeraj Publications www.neerajbooks.com

www.neerajbooks.com

www.neerajbooks.com

if we have the function $f(x) = 4$, then f is constant since f maps any value to 4. More formally, a function $f : A \to B$ is a constant function if $f(x) = f(y)$ for all x and y in A.

2. **Non-Algebric Function:** A function which is not algebraic is called a transcendental function. i.e. $y = pqx$

CONCEPT OF LIMIT

Limit of a Function

The Limit of $f(x)$ as x approaches a is L:

$$\lim_{x\to a} f(x) = L$$

Example:

$$\lim_{x\to -2} f(x) \text{ where } f(x) = \begin{cases} -3x & \text{if } x \neq -2 \\ 1 & \text{if } x = -2 \end{cases}$$

$$\lim_{x\to -2} f(x) = \lim_{x\to -2} -3x$$

$$= -3 \lim_{x\to -2} x$$

$$= -3(-2) = 6$$

Right Hand Limit of a Function

The right-hand limit of $f(x)$, as x approaches a, equals L

$$\lim_{x\to a} f(x) = L$$

Left Hand Limit of a Function

The left-hand limit of $f(x)$, as x approaches a, equals M

$$\lim_{x\to a} f(x) = M$$

Examples of One-Sided Limit

$$\boxed{\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} 2x = 6}$$

$$\lim_{x\to 3^-} f(x)$$

$$\boxed{\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} x^2 = 9}$$

Functions Tending to Infinity:

If the extended real line R is considered, i.e. $R \cup \{-\infty, \infty\}$, then it is possible to define limits of a function at infinity.

If $f(x)$ is a real function, then the limit of f as x approaches infinity is L, denoted

$$\lim_{x\to\infty} f(x) = L,$$

if and only if for all $\varepsilon > 0$ there exists $S > 0$ such that $|f(x) - L| < \varepsilon$ whenever $x > S$.

Similarly, the limit of f as x approaches negative infinity is L, denoted

$$\lim_{x\to -\infty} f(x) = L,$$

if and only if for all $\varepsilon > 0$ there exists $S < 0$ such that $|f(x) - L| < \varepsilon$ whenever $x < S$.

For example

$$\lim_{x\to -\infty} e^x = 0$$

Fundamental Theorems on Limit

If c is any number, $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$, then

(a) $\lim_{x\to a}(f(x) + g(x)) = L + M$

(b) $\lim_{x\to a}(f(x) - g(x)) = L - M$

(c) $\lim_{x\to a}(f(x) . g(x)) = L.M$

(d) $\lim_{x\to a}(f(x) / g(x)) = L/M,\ M \neq 0$

(e) $\lim_{x\to a}(c.f(x)) = c.L$

(f) $\lim_{x\to a} (f(x)n = Ln$

(g) $\lim_{x\to a} c = c$

(h) $\lim_{x\to a} x = a$

(i) $\lim_{x\to a} x^n = a^n$

(j) $\lim_{x\to a}\sqrt{f(x)} = \sqrt{L},\ L > 0$

CONTINUITY

A function f is continuous at the point $x = a$ if the following are true:

(i) $f(a)$ is defined

(ii) $\lim_{x\to a} f(x)$ exists

If f and g are continuous at $x = a$, then

$f \pm g, fg$ and f/g $(g(a) \neq 0)$

are continuous at $x = a$

A polynomial function $y = P(x)$ is continuous at every point x.

A rational function $R(x) = P(x)/Q(x)$ is continuous at every point x in its domain.

Given $f(x) = 3x^2 - 2x - 5$,

Show that $f(x) = 0$ has a solution on $[1, 2]$.

Neeraj Publications www.neerajbooks.com

www.neerajbooks.com

www.neerajbooks.com

$$f(1) = -4 < 0$$
$$f(1) = 3 > 0$$

$f(x)$ is continuous (polynomial) and since $f(1) < 0$ and $f(2) > 0$, by the Intermediate Value Theorem there exists $a\ c$ on $[1, 2]$ such that $f(c) = 0$.

Limits at Infinity

For all $n > 0$,

$$\lim_{x\to\infty} \frac{1}{x^n} = \lim_{x\to-\infty} \frac{1}{x^n} = 0$$

provided that $1/x\ n$ is defined.

CHECK YOUR PROGRESS

Q.1. Distinguish between variable and constants giving examples.

Ans. Variable changes with the change in numerical values whereas constant does not change with their numerical changes.

For Example: variables, *a, b, c, d, e…..*

Constant, π, 1/3, 4/9….

Q. 2. Point out the domain of definition of the following functions:

(i) **$(\cos x + \sin x)/ (\cos x - \sin x) = f(x)$**

Sol. Let $(\cos x + \sin x) / (\cos x - \sin x) = 0$

Domain = Any real value of x except $x = \frac{\pi}{4}$.

(ii) $\sqrt{x^2 - 5x + 6x} = f(x)$

Sol. Let $\sqrt{x^2 - 5x + 6x} = 0$

or, $x^2 - 3x - 2x + 6 = 0;$

or, $x(x - 3) - 2(x - 3) = 0;$

or, $(x - 2)(x - 3) = 0;$

or, $x = 2$ or, $x = 3$

Domain = All real value of x except $2 < x < 3$.

(iii) **$f(x) = \sin^{-1} x$**

Sol. Let $\sin^{-1}x = 0$,

Value of sin lies between -1 and 1

Domain = $-1 < x < 1$

(iv) **$\log(3x - 1) = f(x)$**

Sol. Let $\log(3x - 1) = 0$

or, $3x - 1 = 0$

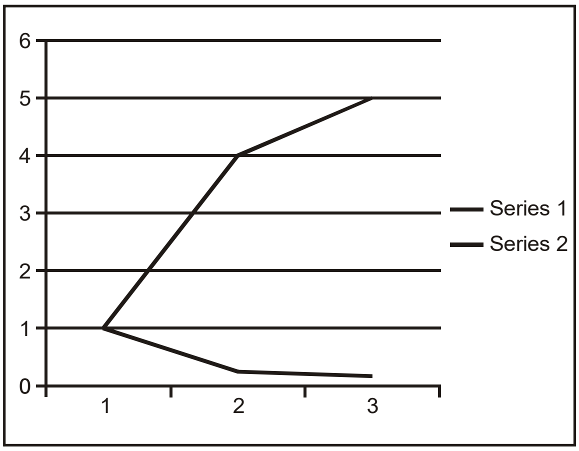
or, $x = 1/3$

Domain = value of $x > 1/3$.

Q. 3. $q = f(p) = 1/p$

Draw the graph of the above function. Show that this is a monotonically decreasing function. Also, show that the function has a lower bound (zero).

Ans.



Q. 4. $f(x) = x^3 + 2x$. Find $\frac{f(x+h) - f(x)}{h}$.

Sol. $\frac{f(x+h) - f(x)}{h} = \frac{f(x+h)}{h} - \frac{f(x)}{h}$

Putting the value of $(x + h)$ at place of x,

$$\frac{f(x+h)}{h} = \{(x+h)^3\}/h + \{2(x+h)\}/h$$
$$= (x^3 + h^3 + 3x^2 h + 3xh^2)/h + (2x + 2h)/h$$
$$= x^3/h + h^2 + 3x^2 + 3xh + 2x/h + 2$$
$$f(x+h) - f(x) = x^3/h + h^2 + 3x^2 + 3xh + 2x/h + 2 - x^3/h - 2x/h$$
$$= 3x^2 + 3xh + h^2 + 2.$$

Q. 5. Evaluate the following limits:

(i) $\lim_{x\to 0}\left(\sqrt{a+x} - \sqrt{a-x}\right)/x$

Sol. $\lim_{x\to 0}(\sqrt{a+x} - \sqrt{a-x})/x$

$$= \lim_{x\to 0}\left(\frac{\sqrt{a+x} - \sqrt{a-x}}{x} . \frac{\sqrt{a+x} + \sqrt{a-x}}{\sqrt{a+x} - \sqrt{a-x}}\right)$$

$$= \lim_{x\to 0}\left(\frac{(\sqrt{a+x})^2 - (\sqrt{a-x})^2}{x} . \frac{1}{\sqrt{a+x} - \sqrt{a-x}}\right)$$

$$= \lim_{x\to 0}\left(\frac{(\sqrt{a+x}) - (\sqrt{a-x})}{x} \cdot \frac{1}{\sqrt{a+x} - \sqrt{a-x}}\right)$$

$$= \lim_{x\to 0}\frac{2x}{x\sqrt{a+x} - \sqrt{a-x}}$$

$$= \lim_{x\to 0}\left(\frac{2}{\sqrt{a+x} - \sqrt{a-x}}\right)$$

Neeraj Publications
www.neerajbooks.com

www.neerajbooks.com