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ELEMENTARY MATHEMATICAL METHODS IN ECONOMICS

■ ■

Sample Preview of the Solved Sample Question Papers

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QUESTION PAPER

(June – 2016)

(Solved)

ELEMENTARY MATHEMATICAL METHODS IN ECONOMICS

Time: 3 hours] *[Maximum Marks: 100*

Note: *Attempt questions from each section as directed.*

SECTION-A

Q. 1. Given the production function:

$y = K^{\alpha}, L^{\beta}$ and Lost function : $c = rK + wL$, find out the minimized cost as function of output level and factor prices. Also make comments on the shape of this function.

Ans. Ref : See Chapter-12, Page No. 93, 'Cost and Supply'.

Concept of Short-run and Long-run

In production function, short-run is defined as period of time when certain inputs cannot be changed. The short- run can be a day, a month or a year. The long-run is defined as period of time when all inputs in the production process are variable.

Long-Run Costs

Suppose, in a production function O = F(L, C), where O = quantity of output produced, L = units of labour employed and C = units of capital employed.

If w is the wage rate per unit of labour and r is rent for unit of capital, the total cost (say M) = wL + rC.

Suppose, total cost remains constant then M = $\overline{M}$, thus $\overline{M} = wL + rC$ …(1)

Or, $$C = \frac{\overline{M} - wL}{r}$$

With equation (1), we can draw an iso-cost line. As shown in the figure (a), the iso-cost line is a downward sloping straight line with slope $\frac{w}{r}$ in the C – L space. There will be different iso-cost lines with different levels of cost outlays. The iso-cost line with highest cost will be farthest from the origin.

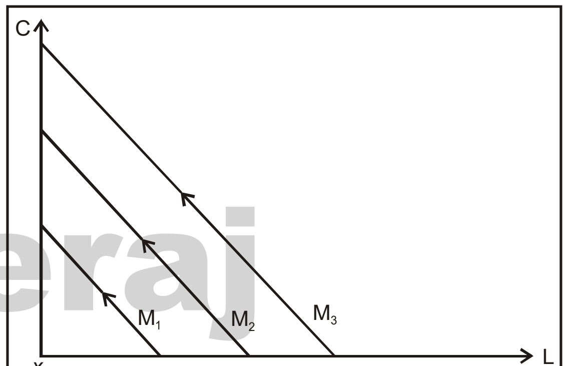

Fig. (a) ISO-cost Lines

As shown in figure (b), the optimal C and L, (provided an output level $\overline{O}$), is obtained from the point of tangency between the lowest iso-cost and the isoquant.

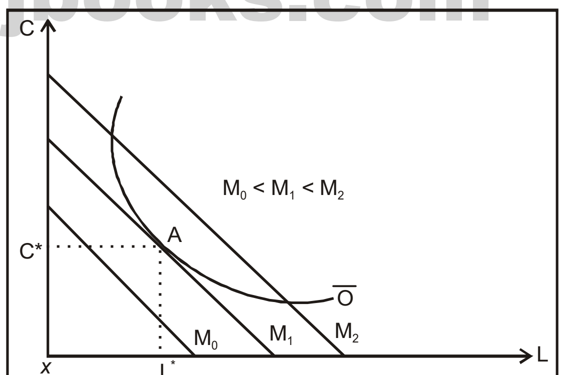

Fig. (b) Optimal Cost-output Conditions

As per the figure, $\overline{O}$ cannot be produced with cost outlay M_0. Besides, M_2 is too costly for producing $\overline{O}$, while M_1 is the optimum cost outlay for production of $\overline{O}$. Also, it can be said that M_1 is the associated cost for output level $\overline{O}$. With M_1 level of cost outlay, $\overline{O}$ is just producible.

However, C and L are variable in the long-run and higher levels of output can be produced. And as we discussed, the optimum cost will be obtained by the tangency between the iso-cost and the iso-quants as given in figure *(c)*.

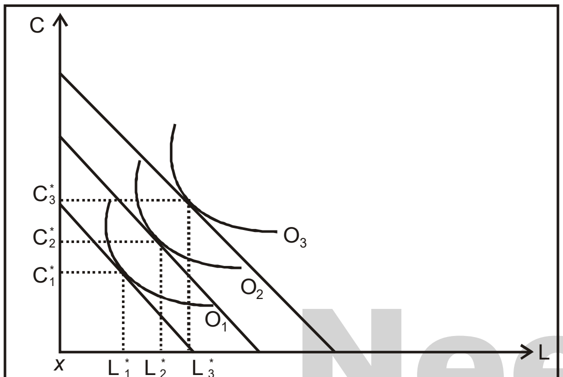

Fig. (c) Map of Optimal Cost-output Combination

Hence, cost is M_1 for output O_1, M_2 for O_2 and M_3 for O_3. From this, we can derive a Long-Run Total Cost Curve (LRTC), which is upward rising. It is shown in the figure *(d)*.

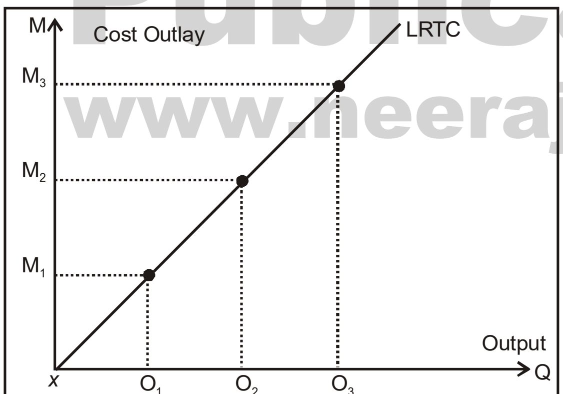

Fig. (d) Long-run Total Cost Curve

Short-Run Costs

In a production process, depending on the time period involved, there may be a number of short-runs. In each short-run, all factors of production cannot be adjusted in the given time period. This will be explained in an example.

Lets take a producer wants to increase output and acquire 90 machines for that. Assume, that there are 3 short-runs each of which is 3 months longer than the previous one. Because of delivery lag, no new machine is added in the first three months. So the producer can have 30 machines in 6 months, 60 in 9 months and 90 in 12 months.

The producer will have to employ different amounts of labour in each of the 3-month period to produce the new output level. Let us assume that man-hours can be adjusted at all times and that the wage rate remains the same.

Let take O_0 as the initial output level and the new higher output is shown by the iso-quant O_1. During the first 3 months, O_1 is produced with $C_0 = 3_0$ and L_1 man-hours and the total cost is given by M_1.

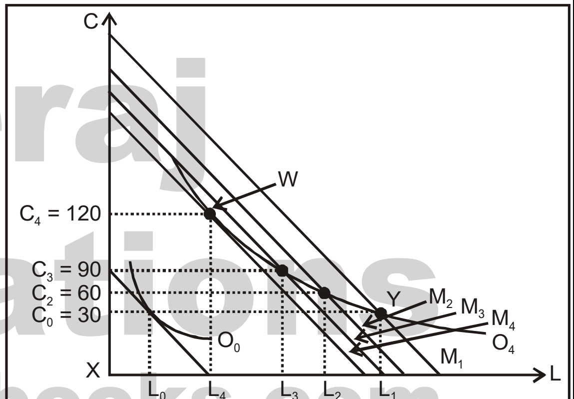

Fig. (e) Short-run Cost

During the next 3 months, 30 new machines are introduced. So $C_2 = 60$. This allows the producer to reduce the labour cost to L_2, as a result total cost falls to M_2. Gradually as new machines are added and the C stock is increased, the entrepreneur is able to produce O_1 optimally at a total cost of M_4, the lowest possible cost at which O_1 can be produced.

In the figure *(f)*, the short-run cost and output is given. Point W is the highest cost incurred and point Y is the lowest cost incurred to produce O_1 level of output. SRTC provides the short-run total cost, when only labour is the variable cost. It is seen that SRTC is greater than LRTC – the long-run total cost when all factors change.

Sample Preview
of
The Chapter

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ELEMENTARY MATHEMATICAL METHODS IN ECONOMICS

ELEMENTARY ALGEBRA

Sets, Relations and Functions

1

INTRODUCTION

Mathematics has been used in the analysis and understanding economic theory. With the basic mathematical language, ideas in economics can be well understood. Identifying and formally defining relationship among primary factors is the first requirement in decision problems, which use mathematical tools. An equation or a set of equations explains these relationships which help the decision-maker in understanding complex economic problems. In this chapter, we will discuss the basic ideas of set, relations and functions.

CHAPTER AT A GLANCE

WHAT IS A SET?

A set is a well defined collection of objects. The elements of a set can be anything: numbers, people, letters of the alphabet and so on. For example, a set of odd numbers (1, 3, 5, 7, 9,…) or a set of alphabets (a, b, c, d, …). A set can have limited as well as unlimited number of entries. Curly bracket is used to write sets with their elements listed in between. For example, a set of even numbers will be written as (2, 4, 6, 8,…). The dots in the example at the end indicates that the set has unlimited or infinity entries.

There are two different ways to specify a set: *(i)* roster method, and *(ii)* descriptive or set builder method.

For example, if 1, 2, 3, 4, 5 and 6 is a set of different product manufactured by a company, by roster method this can be mentioned as:

$$P = \{1, 2, 3, 4, 5, 6\}.$$

In the descriptive method, the rule or condition on the basis of which the elements have been considered as a set is mentioned in curly brackets. So, the set can be mentioned as:

P = {1, 2, 3, 4, 5, 6 are products manufactured by the company}.

Elements of a Set

An element or a member is an entry in a set. Thus, 7 is an element of a set of even numbers. Elements of a set can be written in any order. The elements of a set are mentioned in small letters (a, e, i, o, u), while sets are written in capital letters (P, Q, R, S,...).

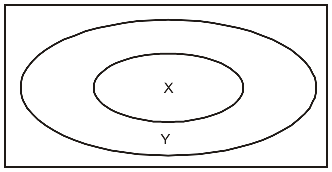
Thus, if a is an element and A is a set, it can be written as $a \in A$. It implies that a belongs to A. If x is not an element of set X, it can be mentioned as $x \notin X$. It means that x does not belong to X. A set without any element is called an empty set. It is mentioned by the Greek letter phi ϕ. Sets can also be elements of a set. In this case, elements are written capital letters A = {X, Y, Z,…}. A is a set of sets X, Y, Z,…

Some Basic Features of Sets

(i) **Equal sets:** If two sets have the same elements, they are equal sets. For example, X = {4,5,6,7} and Y = {4,5,6,7}, they are equal sets. It is mentioned as X = Y.

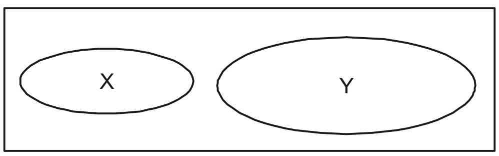
(ii) **Subset:** If the elements of a set X are parts of a set Y, X is a subset of Y. For example, X = {4,5,6,7} and Y = {4,5,6,7, 8, 9, 10}. It will be written as $X \subset Y$ or $Y \supset X$. This implies every element of X is also an element of Y. It also means X is part of Y, or Y includes X, or

X is included in Y, or Y is a superset of X. A set can be subset of itself. Graphically, a subset can be illustrated as given below.



In this figure, X is entirely within Y, or $X \subset Y$. This type of diagram is called Venn diagram.

When two sets (X and Y) have nothing in common, they are written as $X \not\subset Y$ or $Y \not\subset X$. Graphically, it can be presented as given below.



In this figure, X and Y have nothing in common. This is specified as $X \cap Y = \phi$.

(iii) **Proper subset:** If X = {4,5,6,7} and Y = {4, 5, 6, 7, 8, 9, 10}, X is a proper subset of Y. It implies that $X \subset Y$, but $X \neq Y$. Here, X has four elements and Y has seven and $8 \in Y$, but $8 \notin X$.

(iv) **Empty subset:** Set without any element is called empty set. It is mentioned by ϕ. The empty set or null set is written as $\phi = \{ \}$.

(v) **Power set of a set:** Power set is the set of all subsets. For example, if $A_1 = \{a, b\}$, $A_2 = (a, c)$ and $A_3 = \{b, c\}$, the power set of $A = \{(a, b); (a, c); (b, c).\}$. It is mentioned as $P(A) = \{X{:}X \subset A\}$.

If X and Y are two sets,

(i) $X = Y$, if and only $X \subset Y$ and $Y \subset X$

(ii) $X \subset X$ and $\phi \subset Y$

(iii) $X \in P(X)$ and $\phi \in P(X)$

(iv) $P(\phi) = \{\phi\}$. Here $P(\phi)$ is not empty, it has an element ϕ.

A set can have many possible subsets. If $X = \{a, b, c, d, e, f\}$

The possible subsets are:

(i) $X_0 = \{a, b, c, d, e, f\}$

(ii) $X_{10} = \{a, b\}$, $X_{12} = \{b, c\}$, $X_{15} = \{d, e\}$, $X_{20} = \{e, f\}$, $X_{21} = \{a, f\}$, $X_{27} = \{b, e\}$

(iii) $X_{31} = \{a, b, c\}$, $X_{37} = \{c, d, e\}$, $X_{39} = \{d, e, f\}$, $X_{45} = \{a, e, f\}$, $X_{49} = \{a, e, f\}$

(iv) $X_{51} = \{a, b, c, d\}$, $X_{57} = \{b, c, d, e\}$, $X_{59} = \{a, d, e, f\}$, $X_{61} = \{a, b, e, f\}$, $X_{69} = \{a, c, e\}$,

(v) $\phi = \{ \}$.

A set with five elements will have 32 subsets as 2^5. If a set has n elements, a total of 2^n subsets can be formed.

Operations on Sets

Universal set is a union of some sets. The union of sets X, Y, Z can be written as

H = {X, Y, Z}.

Union of sets: The union of set is denoted by $\cup$.

(i) The union of sets X and Y is written as $X \cup Y = \{a \in X \text{ or } a \in Y\}$. a is the element of at least of X and Y. The union of sets X and Y is the set including all elements that belong to X and Y.

(ii) The union of set X of sets is mentioned as $\cup X = \{a \in H{:}\ \exists X \in X\ a \in X\}$. a is the at least one element of X. $\exists$ implies "there exists".

Example 1:

$X = \{a, b, c, d, e, f\}$

$Y = \{e, f, g, h\}$

$X \cup Y = \{a, b, c, d, e, f, g, h\}$.

When sets are united, the common elements are mentioned once only.

Example 2:

X = {all products export by India}

Y = {all products import by Pakistan}

$X \cup Y$ = {all products export by India and all products import by Pakistan}.

Intersection of Sets: The intersection of set is denoted by $\cap$.

(i) The intersection of sets X and Y is written as $X \cap Y = \{a \in H{:}\ a \in X \text{ and } a \in X\}$. a is the common element of both the sets.

(ii) The intersection of a set X is written as $\cap X = \{a \in H{:}\ X \in X \text{ and } a \in X\}$. a belongs to all elements of X. $\forall$ implies "for all".

Example 3:

$X = \{a, b, c, d, e, f\}$

$Y = \{e, f, g, h, i, j\}$

$X \cap Y = \{e, f\}$.

In intersection of sets, the common elements are mentioned once only.

Example 4:

X = {all electronics products produced by Samsung}

Y = {Washing machines produced by Samsung}

$X \cap Y$ = {Washing machines produced by Samsung}.

Example 5:

$X = \{a, b, c, d, e\}$

Y = {g, h, k, l, m}

$X \cap Y = \phi$

This is because they have nothing in common.

Disjoint sets: If X and Y have no common elements $X \cap Y = \phi$, they are called disjoint sets.

Complement of a set: In a universal set (say μ), there are sets (say X, X' and X^c). Here the complement of set X is sets X' and X^c. This is mentioned as X' = {$a \in \mu$ / $a \in X$}.

Difference of sets: It is denoted by "/" or "–".

(i) The difference of sets X and Y is presented as X/Y = {$a \in$ H: $a \in$ X and $a \in$ Y}.
X/Y is also the complement if Y with regard to X.

(ii) H/Y is the complement of Y and is denoted by Y^c = {$a \in$ H: $a \in$ Y}.

Commutativity, associativity and distributivity

(i) The union and intersection of sets are commutative and associative operations:
$X \cup Y = Y \cup X$, $(X \cup Y) \cup Z = X \cup (Y \cup Z)$
$X \cap Y = Y \cap X$, $(X \cap Y) \cap Z = X \cap (Y \cap Z)$

(ii) The union is distributive with regard to the intersection and the intersection is distributive with regard to the union:
$X \cup (Y \cap Z) = (X \cup Y)\ (X \cup Z)$
$X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$

De Morgan's laws

(i) $Z/(X \cup Y) = (Z/X) \cap (Z/Y)$
$Z/(X \cap Y) = (Z/X) \cup (Z/Y)$

(ii) $Z/(\cup X) = \cap \{Z/X: X \in X\}$
$Z/(\cap X) = \cup \{Z/X: X \in X\}$

Ordered Pairs and Cartesian Product of Sets

Ordered pairs: Ordered pair is the set of two separate elements of two sets. For example, A = (a_1, a_2, a_3… a_n) is the weight of staff members in a company and B = (b_1, b_2, b_3… b_n) is the age of the staff. Thus, (a_1, b_1) has the pair of two elements weight and age. If we replace them with number (say $a_1 = 60$ and $b_1 = 29$), the pair will be (60, 29).

If a and b are some objects (say any elements of the basic set H), the ordered pair is presented as (a, b) = {{a},{a,b}}. a and b are the first and second components of the ordered pair. If $a = b$, it will be (a, a) = {{a}}.

Cartesian product of sets: If the ordered pair (60, 29) is changed to (29, 60), there will be a different order. The elements of the sets A and B will differ according to a changed ordering. Thus, the set H of all ordered pairs {a, b} where $a \in$ A and $b \in$ B called the Cartesian product of A and B. The Cartesian product is written as:

$A \times B = \{(a, b) | a \in A \text{ and } b \in B)$

Example:

If A = {1, 2, 3} and B = {a, b}, then
H = {(1, a); (2, a); (3, a); (1, b); (2, b); (3, b)}.
Thus,

(i) (a, b) = (x, y) if $a = x$ and $b = y$.

(ii) The ordered n-tuples (x_1, x_2, …., x_n) and the Cartesian product $X_1 \times X_2 \times \ldots\ldots \times X_n$ ($n \in$ N +, $n > 2$) can be defined in a similar way.

(iii) $X \times X = X^2$, and similarly X^n will be $\underbrace{X \times X \times \ldots \times X}_{n}$.

RELATIONS

A relation is a set of ordered pairs. It is also defined as any subset of a Cartesian product of sets. If A and B are sets and ρ is a relation from A and B, it is written as $\rho \subset A \times B$. If $\rho \subset A \times A$, it means ρ is a relation in A.

If $(a, b) \in \rho$, it is written as $a\ \rho\ b$.

Example: Let X = {2, 4, 8} and Y = {3, 6, 9}. If S is the Cartesian product of X and Y,

S = {(2, 3);(2, 6);(2, 9);(4, 3);(4, 6);(4, 9);(8, 3); (8, 6);(8, 9)}.

To get the relation from S, one of the following conditions can be imposed:

(i) $x < y$
(ii) $(x + y) > 15$
(iii) $(x + y)$ is an exact multiple of 5

For these three cases, the relations are:

(i) (2, 3); (2, 6); (2, 9); (4, 6); (4, 9).
(ii) (8, 9)
(iii) (4, 6)

Examples:

(i) The relation of "less" in
R ρ_1= {$(a, b) \in R \times R : a < b$},
Thus, $(a, b) \in \rho^4 \in R \times R$ if $b - a$ is a positive number.

(ii) The relation of "greater or equal" in
R = {$(a, b) \in R \times R$: $a - b$},
Thus, $(a, b) \in \rho_5 \in R \times R$ if $a - b$ is a positive number.

(iii) The relation between the elements of the set R_0^+ of all positive numbers and the elements of the set T of all triangles of the plane.
ρ_3 = {$(t, x) \in T \times R_0^+$: the area of the triangle t is x}.

(iv) The relation between the elements of the T of all lines of the plane and the elements of the set C of all circles of the plane and, $\rho_3 = \{(c, l) \in C \times L: l$ is triangle of $c\}$.

Domain and Range of Relations

If A and B are sets and ρ is a relation from A to B ($\rho \subset A \times B$)

The domain of ρ will be $D(\rho) := \{a \in A : \exists\, b \in B(a, b) \in \rho\}$.

The range of ρ will be $R(\rho) := \{b \in B : \exists a \in A(a, b) \in \rho\}$.

Properties of Relations

Suppose A is a set and ρ is a relation in A ($\rho \subset A^2$). We are talking about a subset of a Cartesian product of set A with itself.

(i) **Reflexivity**

ρ is reflexive if $\forall\, a \in A(a, a) \in \rho$. ($\forall$ means "for all").

(ii) **Irreflexivity**

ρ is irreflexive if $\forall\, a \in A(a, a) \in \rho$.

(iii) **Symmetry**

ρ is symmetric if $\forall\,(a, b) \in \rho\ (b, a) \in \rho$, $(a, b) \in \rho$ implies $(b, a) \in \rho$.

(iv) **Antisymmetry**

ρ is antisymmetric if $((a, b) \in \rho$ and $(b, a) \in \rho)$ shows $a = b$.

(v) **Transitivity**

ρ is transitive if $((a, b) \in \rho$ and $(b, c) \in \rho)$ shows $(a, c) \in \rho$.

Special Relations

Suppose A is a nonempty set and ρ is a relation in $A(\rho \subset A^2)$. We are talking about a subset of a Cartesian product of set A with itself.

(i) **Equivalence relation:** ρ will be in an equivalence relation if it is reflexive, symmetric and transitive.

(ii) **Order relation:** ρ will be in an order relation if it is reflexive, antisymmetric and transitive. ρ is a total (linear) order relation if for each $(a, b) \in A^2\ (a, b) \in \rho$ or $(b, a) \in \rho$ is satisfied; If it is not, ρ will be a partial order relation.

Inverse relation: Suppose A and B are sets and is a relation from A and B.

The inverse of ρ (mentioned as ρ^{-1}) is presented as $\rho^{-1} = \{(b, a) \in B \times A : (a, b) \in \rho\}$

Classification of sets: If A is a nonempty set. A set X (of subsets of A) is called a classification of A if the properties given below are satisfied:

(i) $\forall\, X \in X$. X is nonempty subset of A.

(ii) $(X, Y \in X$ and $X \neq Y)$ means $X \cap Y = \phi$

(iii) $\cup X = A$.

The elements of X are the classes of the classification.

Equivalence classes: Suppose A is a nonempty set and is an equivalence relation in A. Here for each $a \in A$, the equivalence relation is defined as $X_a := \{b \in A : (a, b) \in \rho\} \in P(A)$. X_a is the equivalence class of a.

FUNCTIONS

Function is rule of associating an element of one set with an element of another set. If the relation exists such that for each value of x there exists one corresponding value of y, then y is said to be a function of x. Suppose $y = (x)^2$, then for the value of $x = 2$, then $y = 4$. However, in ordered pair such a function is not always possible.

For example, if X = {1, 2, 8} and Y = {3, 6, 9}, in subsets, the single value of 8 will be associated with 3, 6, and 9. So it is not always possible to determine the value of y.

If y is said to be a function of x, it is denoted as $y = f(x)$.

A function is also called a transformation. It is denoted by $f : X \rightarrow Y$, $x \in X$ and $y \in Y$.

Suppose X and Y are two sets, a relation $f \subset X \times Y$ is referred to as a function from X to Y if *(i)* $D(f) = X$, and *(ii)* the set has exactly one element.

In a function $y = f(x)$, x is called the argument of the function or independent variable and y is called the value of the function or dependent variable.

Variable: A variable is a representation of a number, which may take different numerical values in mathematics. A variable can assume various values. The magnitude of variables varies. They are thus represented by symbols instead of specific numbers. In applied mathematics, *a* variable is represented by the first letter of its name. For example, *d* for demand, *q* for quantity, *p* for price and *s* for sales and savings.

Variables can be categorized in various ways. A variable can be discrete (countable with numbers – 100 machines or 1000 employees) or continuous (measurable as 20 degree centigrade temperature or 2 feet and 2 inches height).