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By: *Manisha Sharma*

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Content

ECONOMETRIC METHODS

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QUESTION PAPER

June – 2023

(Solved)

ECONOMETRIC METHODS

M.E.C.E.-1

Time: 3 Hours] ***[Maximum Marks: 100***

***Note:** Attempt questions from each section as per instructions given.*

SECTION - A

Note: Answer any *two* questions from this Section.

Q. 1. For the regression equation $Y = \alpha + \beta X + \varepsilon$, explain the procedure of estimation of the parameters. If an estimated equation is given as:

$$Y = 1.5 + 0.37X_1 - 0.5X_2$$
$$(1.7) \quad (0.21) \quad (1.5)$$

Interpret the results and test the significance of the estimates. (Figures in parentheses below indicate standard error).

Ans. In the specification of the model as described by eq. $Y = \alpha + \beta X + U$, the values of the parameters *a* and *b* are not known, as a result the population regression line is not known. When the values of a and *b* are estimated, we obtain a sample regression line that serves as an estimate of the population regression line.

If *a* and *b* are estimated by $\hat{\alpha}$ and $\hat{\beta}$ respectively, then the sample regression line is given by,

$$\hat{Y}_i = \hat{\alpha} + \hat{\beta} X_i$$

Where $\hat{Y}_i$ is the fitted value of Y_i.

The theory of estimated can be divided into two parts–point estimation and interval estimation. In point estimation, the aim is to use the prior and the sample information for the purpose of calculating a value which would be, in some sense, our best guess as to the actual value of the parameter of interest. In interval estimation, on the other hand, the same information is used for the purpose of producing an interval which would, contain the true value of the parameter with some given level of probability. Since an interval is fully characterised by its limits, estimating an interval is equivalent to estimating its limits. The interval itself is usually called a confidence interval. Confidence intervals can also be viewed as possible measures of the precision of a point estimator. The problem of point estimation is that of producing an estimate that would represent our best guess about the value of the parameter. To solve this problem, we have to do 'two things. First, to specify what we mean by 'best guess' and second, to devise estimators that would meet this criterion. The first part of the problem amounts to specifying various properties of an estimator that can be considered desirable. The properties of the estimators are: unbiasedness, efficiency, minimum mean sqwe error and consistency. The second part, on the other hand, involves devising estimators that would have at least some of the desirable properties. The estimators are given names that indicate. Nature of the principle used in devising the formula.

$$Y = 1.5 + 0.37\, X_1 - 0.5\, X_2$$
$$(1.7) \quad (0.21) \quad (1.5)$$

Where the value in parentheses is the standard error of the coefficient.

from this *eq.* we can compute

Let,

$$RSS = \Sigma\hat{Y}^2 = 1383.16$$
$$TSS = \Sigma Y_2 = 1480.00$$

given values $\hat{\beta}_1 = 0.37$

$$\hat{\beta}_2 = -0.5$$
$$\hat{\alpha} = 5.85$$

So that

$$^s\alpha = 1.7$$
$$^s\beta_1 = 0.21$$
$$^s\beta_2 = 1.5$$

$$R_2 = \frac{RSS}{TSS} = \frac{13.83.16}{14.80.00}$$
$$= 0.93$$

Finaly, the test of significance of R^2 is provided by F ratio.

$$F_{(2,2)} = \frac{RSS/(k-1)}{Ess/(n-k)} = 14.28$$

This F ratio is a test of significance of both explanatory variable simultamousty. Here it is significant at 10% stand of significance as the conputed values (14.18) exceesses the critical value (9.0) as σ 10% level of significance.

Q. 2. Explain the concept of autocorrelation. What are its consequences on the OLS estimates? Explain any one of the methods of detecting autocorrelation in a regression model.

Ans. Ref.: See Chapter-7, Page No. 52, 'Introduction', 'Presence of Autocorrelation', Page No. 55, 'Testing for Autocorrelation'.

Q. 3. When do we use the method of Generalised Least Squares (GLS)? Outline the procedure of finding GLS estimator for the model

$$Y = X\beta + U.$$

Ans. The Generalized Least Squares (GLS) method is used in regression analysis when the assumptions of the classical Ordinary Least Squares (OLS) regression are violated. Specifically, GLS is employed under the following circumstances

Heteroscedasticity: When the variance of the errors (residuals) is not constant across all levels of the independent variables, GLS can be used to correct for this. Heteroscedasticity violates one of the OLS assumptions, which assumes constant variance of errors.

Autocorrelation: In time-series data or panel data where observations are taken over time, errors can be correlated with each-other. GLS can be applied to account for autocorrelation, where the errors are correlated across different time periods.

Panel Data: When dealing with panel data (data collected on the same set of individuals, firms, or regions over time), GLS is often used to account for both heteroscedasticity and autocorrelation within panels.

Clustered Data: In the presence of clustered or grouped data, where observations within the same cluster are likely to be correlated, GLS can be used to adjust for this correlation.

Measurement Errors: When there are errors in the measurement of variables, GLS can be used to handle measurement errors in both the dependent and independent variables.

Endogeneity: GLS can be applied when there is endogeneity in the model, meaning that some independent variables are correlated with the error term.

Procedure for Finding GLS Estimator:

1. Assumptions:

- Assume a linear regression model $Y = X\beta+U$, where Y is the dependent variable, X is the matrix of independent variables, β is the vector of parameters to be estimated, and U is the error term.
- Assume that the errors U have a covariance matrix $\Sigma\Sigma$ that is not diagonal (indicating heteroscedasticity and/or autocorrelation).

2. Weighting Matrix (W):

- Calculate the inverse of the covariance matrix of the errors, denoted as $= \Sigma - 1$ $W = \Sigma - 1$. This matrix captures the relationships between the errors.

3. Transform the Model:

- Pre-multiply the entire model by $W^{0.5}$ on both sides. This transformation results in a new model where the errors have constant variance and are uncorrelated.

$$W^{0.5}Y = W^{0.5}X\beta + W^{0.5}U^*.$$

Here, U* represents the transformed errors with constant variance and no autocorrelation.

4. Ordinary Least Squares (OLS) Estimation:

- Apply OLS to the transformed model: $W^{0.5}Y = W^{0.5}X\beta + W^{0.5}U^*$.
- Estimate the parameters β using OLS.

5. GLS Estimation:

- The GLS estimator for the original model is given by $\beta\text{\^{}}GLS = (X'WX)^{-1}X'WY$.

Q. 4. Explain the underlying ideas behind the logit model. Explain on what grounds the logit model is an improvement over the linear probability model.

Ans. Ref.: See Chapter-12, Page No. 98, 'The Logit Model'.

SECTION-B

Note: Answer any *five* questions from this Section.

Q. 5. What is meant by heteroscedasticity? What are its effects on the following?

Ans. Ref.: See Chapter-8, Page No. 62, 'Sources of Heteroscedasticity'.

***(a)* OLS estimators and their variances.**

Ans. Ref.: See Chapter-8, Page No. 62, 'Consequences'.

***(b)* Confidence Intervals.**

Ans. Ref.: See Chapter-3, Page No. 19, 'Confidence Intervals'.

***(c)* The use of *t* and F test of significance.**

Ans. Ref.: See Chapter-1, Page No. 3, 'The student's–*t* Distribution' and Page No. 4, 'F–Distribution'.

Sample Preview
of
The Chapter

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ECONOMETRIC METHODS

BASIC ECONOMETRIC THEORY

Introduction to Econometrics

1

INTRODUCTION

Econometrics is the result of a certain outlook on the role of economics, consists of the application of mathematical statistics to economic data to lend empirical support to the models constructed by mathematical economics and to obtain numerical results.

Economics has emerged as a specialized branch of economics. It is may be defined as the social science in which the tools of economic theory, mathematics and statistical inference are applied to the analysis of economic phenomena.

Today, we would say that econometrics is the unified study of economic models mathematical statistics, and economic data within the field of econometrics there are sub-divisions and specialization econometric theory concerns the development of tools and methods and the study of the properties of econometric methods.

Applied econometric is a term describing the development of quantitative economic models and the application of econometric methods of these models using economic data.

CHAPTER AT A GLANCE

THE NATURE OF ECONOMETRICS

Econometrics means, "Economic Measurement". Although measurement is an important part of econometrics, the scope of econometrics is much broaden.

Economic theory makes statements or hypotheses that are mostly qualitative in nature. For example, microeconomic theory states that, other things remaining the same, a reduction in the price of a commodity is expected to increase the quantity demanded of that commodity. Thus, economic theory postulates a negative or inverse relationship between the price and quantity demanded of a commodity. But the theory itself does not provide any numerical measure of the relationship between the two *i.e.*, it does not tell by how much the quantity will go up or down as a result of a certain change in the price of the commodity.

The main concern of mathematical economics is to express economic theory in mathematical form with out regard to measurability or mainly interested in the theory. Econometric is noted previously, is mainly interested in the empirical verification of economic theory. As we shall see the econometrician often uses the mathematical equations proposed by the mathematical economist puts these equations in such a form that they lend themselves to empirical testing. And this conversion of mathematical into econometric equations requires a great deal of ingenuity and practical skill.

The traditional or classical methodology, which still dominates empirical reaserch in economics and other social and behavioural sciences. Traditional econometric methodology proceeds along the following lines:

- Statement of theory of hypotheis.
- Specification of the mathematical model of the theory.
- Specification of the statistical, or econometric model.

- Obtaining the data.
- Estimation of the parameters of the econometric model.
- Hypothesis testing.
- Using the model for control or policy purpose.

PROBABILITY DISTRIBUTION

Galileo was the first Italian mathematician, who attempt at a quantitative measure of probability while dealing with some problems related to the theory of dice in gambling. But the first foundation of the mathematical theory of probability was laid in the mid-seventeenth century by two French mathematicians B Pascal and P Fermat, while solving a number of problems by French gambler and noble man Chevalier De-Mere to Pascal.

Russian mathematician also have made very valuable contributions to the modern theory of probability. In this section we shall explain the various term which are used in the definition of probability under different approaches.

Random Experiment: If in each trial of an experiment conducted under identical conditions, the outcome is not unique, but may be any one of the possible outcomes then such an experiement is called a random experiment.

Outcome: The result of a random experiment will be called an outcome.

Trial and Event: Any particular performance of a random experiment is called a trial and outcome or combination of outcomes are termed as events.

If the total number of possible outcomes of a random experiment is known as the exhaustive events or case.

If the number of cases favourable to an event in a trial is the number of outcomes which entail the happening of the event is called favourable events or cases.

Mutually Exclusive Events: Events are said to be mutually exclusive or incompatible if the happening of any one of them precudes the happening of all the other *i.e.*, if no two or more of them can happen simultaneously in the same trial.

Probability Function: P(A) is the probability function defined on a field B of events if the following properties exists:

(i) $\forall$ A $\in$ B, P(A) $\geq$ 0. It is called property of non-negativity.

(ii) P(S) = 1. It is called property of certainty.

(iii) If $\{A_n\}$ is any finite or infinite sequence of disjoint events in B, then

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i).$$

It is called property of additivity.

Distribution Function

Let x be a random variable. The function F defined $\forall\ x$ by $F(x) = P(X \leq x) = P\{\omega : X(\omega) \leq \omega\}, -\infty < x < \infty$ is called the distribution function of the random variable.

Remark: *(i)* If F is the distribution function of random variable X and if $a < b$, then

$P(a < x \leq b) = F(b) - F(a)$

(ii) If F is the distribution function of one dimensional random variable X then.

$\rightarrow 0 \leq F(x) \leq 1$

$\rightarrow F(x) \leq F(y)$ if $x < y$

(iii) All distribution functions are monotonically non-decreasing and lie between 0 and 1.

Discrete Probability Distribution

A real valued function defined on a discrete sample space is called a discrete random variable.

If X is a discrete random variable with distinct values $x_1, x_2, ..., x_n, ...$ then the function

$$P(x) = \begin{cases} P(X = x_1) = P_1 & \text{if } x = x_i \\ 0 & \text{if } x \neq x_i \end{cases}$$

Where, $i = 1, 2, 3, ...$

Remark: *(i)* $0 \leq F(x) \leq 1, -\infty < x < \infty$

(ii) F (x) is non-decreasing function of x.

(iii) F (x) is continuous function of x on the right.

(iv) The discontinuity of F(x) are at the most countable.

Binomial Distribution

Binomial distribution was discovered by James Bernoulli in the year 1700 and was first published posthumously in 1713, eight year after his death.

A random variable X is said to follow binomial distribution if it assumes only non-negative values and its probability mass function is given by:

$$P(X = x) = P(x) = \begin{cases} \binom{n}{x} p^x q^{n-x}; & x = 0, 1, 2, ...; q = 1 - p \\ 0 & \text{Otherwise} \end{cases}$$

Remark: *(i)* The two independent constants n and p in the distributions are known as the parameters of the distribution. 'n' in also sometimes known as the degree of the binomial distribution.

(ii) This experiment consists of a sequence of n repeated trials.

(iii) Each trial result in an outcome that may be classified either as a success or a failure is called the probability mass function of random variable of X.

Remark: *(i)* Here, the number $p(x_i)$; $i = 1, 2, 3, ...$ must satisfy the following conditions:

(a) $P(x_i) \geq 0 \;\forall\; i$

(b) $\sum_{i=1}^{\infty} P(x_i) = 1$

(ii) The set of values which X takes is called the spectrum of the random variable.

Continuous Probability Distribution

A random variable is said to be continuous when its different values cannot be put in one-one correspondence with a set of positive integers. A continuous random variable is a random variable that (at least conceptually) can be measured to any desired degree of accuracy.

If X is a continuous random variable with the p.d.f. $f(x)$, then the function $F_X(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)\, dt$; $-\infty < x < \infty$ is called the continuous distribution function.

Normal Distribution: The normal distribution was first distribution in 1733 by English mathematician De-Moivre who obtained this continuous distribution as a limiting case of the binomial distribution and applied it to problems arising in the game of chance.

A random variable X is said to have a normal distribution with parameter μ and σ^2, if its p.d.f. is given by the probability law:

$$f(x\,;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}$$

$$\Rightarrow f(x\,;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/\sigma^2}\;; -\infty < x < \infty,$$

$-\infty < \mu < \infty$, $\sigma > 0$

Where μ = Mean

σ^2 = Variance

Remark: (i) When random variable is normally distribution with mean μ and standard deviation σ, it is customary to write X is distributed as N (μ, σ^2) *i.e.*, $X \sim N(\mu, \sigma^2)$.

(ii) The p.d.f. of standard normal variable Z is given by:

$$\varphi(Z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}, \; -\infty < z < \infty$$ and the corresponding distribution function, defined by $\phi(z)$ is given by:

$$\varphi(Z) = P(Z \leq z) = \int_{-z}^{z} \varphi(u)\, du = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\mu^2/2}\, du$$

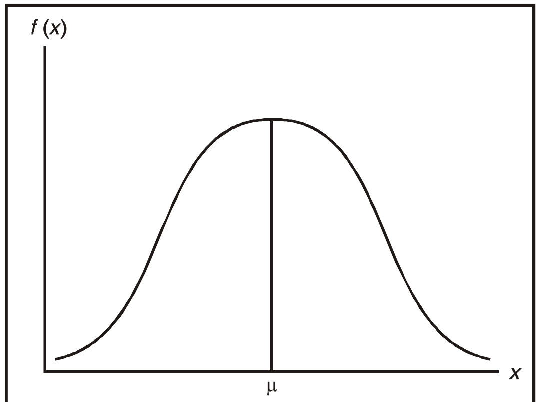

Fig.: Normal Curve

The Student's-*t* Distribution

W.S. Gosset, who wrote under Pseudonym (pen-name) of student defined his t in a slightly different way viz, $t = (\bar{x} - \mu)/s$ and investigated its sampling distribution, some what empirically in a paper entitled *'The Probable Error of the Mean'* published in 1908. Professor R.A. Fisher later on defined his own *'t'* and gave a rigorous proof for its sampling distribution in 1926. The salient feature of '*t*' is that both the statistic and its sampling distribution are functionally independent of σ, the population standard deviation.

Let x_i (i = 1, 2, 3, ..., n) be a random sample of size n from a normal population with μ and variance σ^2. There the student's – t distribution is defined by:

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

Where $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$, is the sample mean and

$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$, is an unbiased estimate of the population variance σ^2, and it follows student's t-distribution with = $(n - 1)$ *d. f.* with probability density function.

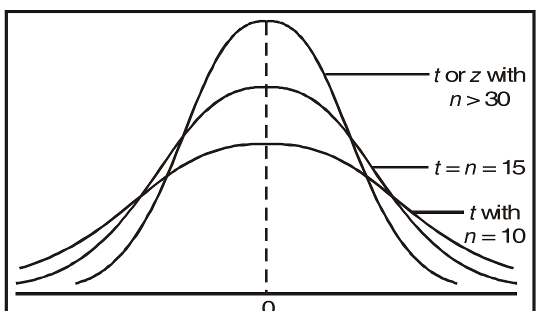

Fig. Student's *t*-Distribution

Chi-Square Distribution

The square of a standard normal variate is known as a Chi-square variate with degree of freedom is 'one'.

Thus if $X \sim N(\mu, \sigma^2)$, then $z = \frac{X-\mu}{\sigma} \sim N(0,1)$ and $Z^2 = \left(\frac{X-\mu}{\sigma}\right)^2$ is a Chi-square variate with 'one' degree of freedom.

In general if X_i, ($i = 1, 2, 3, ..., n$) are n independent normal variates with means μ and variance σ_i^2, ($i = 1, 2, 3, ...,n$) then $\chi^2 = \sum_{i=1}^{n}\left(\frac{X_i-\mu_i}{\sigma_i}\right)^2$, is a Chi-square variate with *'n'* degree of freedom.

Remark: *(i)* If a random variable has a Chi-square distribution with degree of freedom *'n'* we write $X \sim \chi^2_{(n)}$ and its p.d.f.

$$f(x) = \frac{1}{2^{n/2}\sqrt{(n/2)}} e^{-x/2} x^{(n-2)-1} ; 0 \le x < \infty$$

(ii) If $X \sim \chi^2_{(n)}$, then $\frac{1}{2}X \sim \chi\left(\frac{1}{2}n\right)$

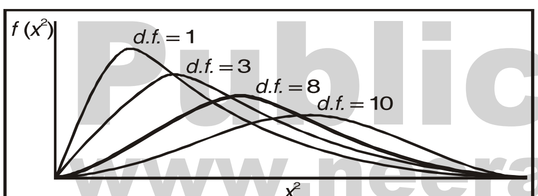

Fig. Chi-Square Distribution

F-Distribution: If Z_1 and Z_2 are two independent Chi-square varites with k_1 and k_2 d.f. respectively, then F-statistic is defined by:

$$F = \frac{Z_1/k_1}{Z_2/k_2}$$

In other words, F is defined as the ratio of two independent Chi-square variates divided by their respective degree of freedom and it follows Snedecor's F-distribution with (k_1, k_2) *d.f.* with probability function given by:

$$f(F) = \frac{\left(\frac{k_1}{k_2}\right)^{k_1/2}}{B\left(\frac{k_1}{2}, \frac{k_2}{2}\right)} \cdot \frac{F^{\frac{k_1}{2}-1}}{\left(1+\frac{k_1}{k_2}F\right)^{(k_1+k_2)/2}}, \quad 0 \le F < \infty .$$

Remark: *(i)* A stastistic F following Snedecor's F-distribution with (k_1, k_2) d.f. will be denoted by $F \sim F(k_1, k_2)$

(ii) The sampling F-distribution does not involve any population parameters and depends only on the degree of freedom k_1 and k_2.

SAMPLING DISTRIBUTION

Before giving the notation of sampling we will first define population. In a econometric investigation the interest usually lies in the assessment of the general magnitude and the study of variation with respect to one or more characteristics relating to individuals belonging to a group. This group of individuals under study is called population or universe. Thus, in statistics population is an aggregate or objects animate or inanimate under study. The population may be finite or infinite.

If the population is infinite complete enumeration is not possible. Also if the units are destroyed in the course of inspection , 100% inspection, though possible is not at all desirable. But even if the population is finite on the inspection is not distructive, 100% inspection is not taken recourse to because of multplicity of causes, viz, administrative and financial implication, time factor, etc., and we take the help of sampling.

A finite subset of statistical individuals in a population is called a sample and the number of individual in a sample is called the sample size.

For the purpose of determining population characteristics instead of enumerating the entire population, the individuals in the sample only are observed. Then the sample characteristics are utilised to approximately determine or estimate the population. For example, on examining the sample of a particular stuff we arrive at a decision of purchasing or rejecting that stuff. The error involved in such approximation is known as sampling error and is inherent and unavoidable in any and every sampling scheme.

There are four types of sampling is:

(i) Purposive Sampling *(ii)* Random Sampling.

(iii) Startified Sampling *(iv)* Systematic Sampling.

STATISTICAL INFERENCE

Let us consider a random variable X with p.d.f. $f(x, \theta)$. In most common applications, though not always, the function form of the population distribution is assumed to be known except for the value of some unknown parameter (s) θ which may take any value on a set Θ. This is expressed by writing the p.d.f in the form $f(x, \theta)$, $\theta \in \Theta$. The set Θ, which is the set of all possible values of θ is called the parameter space.